

Extensions and applications of the Isoperimetric Inequalities

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In this lecture, we will explore several extensions and applications of the classical isoperimetric inequalities. Our approach will incorporate general measures and fractional Poincaré-type estimates, including the correction factor developed by Bourgain, Brezis, and Mironescu. Next, we will consider extensions of the classical Sobolev-type subrepresentation formula:

$$|f(x)| \leq c_n I_1(|\nabla f|)(x)$$

Here, I_1 denotes the classical fractional integral operator. We will extend this formula in two different ways: by improving the right-hand side with fractional derivatives in place of the gradient $|\nabla|$, and by replacing the identity operator on the left with singular integral operators.

The main part of the lecture is based on joint work with Cong Hoang and Kabe Moen.

References

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