ABSTRACTS

Harmonic Analysis and its Applications in Matsumoto 2016, winter

15th - 19th, February 2016

at

Shinshu University (Matsumoto Campus)

Joachim Toft (Linnaeus University, Sweden)
Harmonic analysis on Fourier invariant small test function and large distribution spaces

Abstract

The series of seminars consists of three lectures:

Lecture 1

We consider a family of function spaces which include all Fourier Invariant Gelfand-Shilov spaces (or FIGS-spaces) of standard type, and their distribution spaces. The set of finite linear combinations of the Hermite functions is dense in each such space. Moreover, the smallest spaces in the family are significantly smaller than the non-trivial FIGS-spaces, giving that the family of corresponding distribution spaces contain spaces which are larger than any Fourier invariant Gelfand-Shilov distribution space. The spaces in the family are defined by imposing suitable estimates when applying powers of the harmonic oscillator on the functions, an approach, introduced by S. Pilipović in the 80'. For this reason we call any space in the family as *Pilipović space*. We consider several basic properties on these spaces. For example:

- We explain characterizations of such spaces in terms of Hermite series expansions and describe their images under the Bargmann transform.
- A broad class of modulation spaces is introduced, parameterized among others with weight functions ω only obeying the conditions that $\omega, \frac{1}{\omega} \in L^\infty_{loc}$. We show that these modulation spaces are complete.
- We deduce that any for any operator T with kernel $K(x;y)$ in a specific Pilipović space can be factorized into a product of two operators with kernels in the same Pilipović space, $x;y \in \mathbf{R}^d$. Furthermore, the factorization can be performed in such way that the L^2-eigenfunctions for one of the operators are the Hermite functions. We use these facts to deduce that the singular values of T obey estimates of the form

$$\sigma_j(T) \lesssim e^{-c \cdot j^{\frac{1}{2ds}}}$$

Parts of the first lecture is based on joint work with Y. Chen and M Signahl.

Lecture 2

In the second lecture we continue the discussions on modulation spaces. In comparison to the first lecture, we assume that the weight functions should be moderate, giving that our spaces stay between the Gelfand-Shilov space $\sum_1$ and its distribution space. We present recent results on composition, continuity and Schatten-von Neumann (SvN) properties for operators and pseudo-differential operators (ΦDOs) when acting on modulation spaces. For example we present necessary and sufficient conditions in order for the Weyl product should be continuous on modulation spaces. Such question is strongly connected to questions wether compositions of ΦDOs with symbols in modulation spaces remain as ΦDOs with a symbol in a modulation space. We also present necessary and sufficient conditions for ΦDOs with symbols in modulation spaces to be SvN operators of certain degree in the interval $(0, 1]$. Note that, so far, only few results on SvN operators with degrees less than one are available in the literature.

Parts of the second lecture is based on joint work with Y. Chen, E. Cordero and P. Wahlberg.

Lecture 3

In the third lecture we consider pseudo-differential operators, with symbols which are allowed to grow super exponentially, but at the same time obeying strong regularity conditions. For such operators se deduce basic continuity on Gelfand-Shilov spaces and their distribution spaces. As special case we also consider the harmonic oscillator, and perform some further studies of the Weyl symbol of its inverse. In particular, we show that its inverse fits well in the classes of the family of pseudo-differential operators described above.

The third lecture is based on joint works with M. Cappiello and L. Rodino.

Oliver Dragičević (University of Ljubljana, Slovenia)

Bilinear embedding, heat flow and Bellman functions

Abstract

We discuss a particular kind of estimates, which we call "bilinear embeddings of Littlewood-Paley type", their proofs and implications. Our approach consists of analyzing the flow associated with the semigroups generated by the Laplacians in question, to which suitably chosen functions are applied. One of the features in all our proofs is a particular function invented by F.L. Nazarov and S.R. Treil in 1995. We find and exploit convexity-type properties of this function which, together with its size estimates, justify one in calling it a "Bellman function".

Earlier applications of the heat-flow technique merged with Bellman functions involved estimates of Riesz transforms corresponding to various Laplacians. The scope of the method has been considerably extended since. In our illustrations we describe in detail two recent examples:

i) optimal holomorphic functional calculus for generators of symmetric contraction semigroups, and

ii) bilinear embedding for elliptic operators in divergence form with complex coefficients.

The talks are based on joint works with Andrea Carbonaro (U. of Genova).

Camil Muscalu (Cornell University, US)

Iterated Fourier series

Abstract

The Plan of the lectures is to describe the theory of *iterated Fourier series* and its connections to other parts of mathematics.

Jonathan Bennett (University of Birmingham, U.K.)

Weighted Harmonic analysis beyond Calderón-Zygmund theory

Abstract

At the Williamstown conference on Harmonic Analysis in 1978, E. M. Stein raised the possibility that the disc multiplier operator might be controlled by the "universal maximal function" via a weighted norm inequality. More specifically, Stein asked whether an inequality of the form

$$\int_{\mathbb{R}^n} |Sf|^2 w \lesssim \int_{\mathbb{R}^d} |f|^2 \mathcal{M} w$$

could hold, where S is the disc multiplier operator, and $\mathcal{M}$ is some variant of the Hardy-Littlewood maximal function involving averages over arbitrary rectangles. Questions with this flavour in the classical context of Calderón-Zygmund singular integral operators, and their variants, have been the subject of intensive study in recent decades, where a very fine understanding has emerged (a particular culmination being the resolution of the A_2 conjecture). However, despite serious attempts, Stein's question remains far from resolved even in two dimensions. Of course the disc multiplier operator, in dimensions $n > 1$, is very far from being a Calderón-Zygmund operator - its convolution kernel

$$K(x) = c\frac{e^{2\pi i|\xi|} + e^{-2\pi i|\xi|} + o(1)}{|\xi|^{\frac{n+1}{2}}}$$

is oscillatory in nature and is far from being integrable.

The purpose of these lectures is to begin to develop a weighted theory which is able to function in such highly oscillatory contexts; a simple example we might want to think of could be convolution with $e^{i|x|^3}$ on $\mathbb{R}^n$. For natural classes of such operators (inspired by work of A. Miyachi from the 1980s) we identify the maximal operators $\mathcal{M}$ which control them via L^2 weighted inequalities of the above type. These maximal operators turn out to be strongly geometric, involving fractional averages and "approach regions" in $\mathbb{R}^n \times \mathbb{R}_+$, and will be accessed via singular variants of the classical Littlewood–Paley–Stein square functions. As we shall see, such inequalities for oscillatory operators have the additional virtue of applying in the context of dispersive PDE, such as the time-dependent Schrödinger equation on $\mathbb{R}^n$.

Neal Bez (Saitama University, Japan)

Applications of the Funk-Hecke theorem to smoothing and trace estimates

Abstract

For Kato-smoothing estimates with radially symmetric weights, I will show how to use the Funk-Hecke theorem to generate a new expression for the optimal constant involving the Fourier transform of the weight. Several applications will be presented, including a quickly deducible theorem which unifies many well-studied smoothing estimates, along with the sharpness of decay and smoothness exponents. Connections and applications to the Mizohata-Takeuchi conjecture and trace estimates for the sphere will also be discussed. This will be an exposition of recent joint work with Hiroki Saito (Kogakuin) and Mitsuru Sugimoto (Nagoya).

Jayson Cunanan (Shinshu University, Japan)

Inclusion relations and trace theorems of Wiener amalgam spaces

Abstract

We discuss inclusion relations between Wiener amalgam spaces and L^p-Sobolev space. We also discuss the trace operator $f \to f(\cdot, 0)$ acting on (standard) Wiener amalgam spaces and its anisotropic versions. The optimality of these results are also established. This talk is based on collaborated works with M. Kobayashi, M. Sugimoto and Y. Tsutsui.

Naohito Tomita (Osaka University, Japan)

On the boundedness of multilinear Fourier multiplier operators

Abstract

This talk has two purposes. One is to discuss the boundedness of trilinear flag paraproducts. The second purpose is to discuss the regularity conditions for multilinear Fourier multipliers to assure the boundedness of the corresponding operators. The first part is based on a joint work with Professor Akihiko Miyachi and the second part on a joint work with Professors Loukas Grafakos, Akihiko Miyachi and Hanh Van Nguyen.

Hidemitsu Wadade (Kanazawa University, Japan)

Scaling invariant Hardy inequalities of multiple logarithmic type on the whole space

Abstract

In this talk, we consider Hardy inequalities of logarithmic type involving singularities on spheres in the whole space in terms of the Sobolev-Lorentz-Zygmund spaces. We prove it by absorbing singularities of functions on the spheres by subtracting the corresponding limiting values. We also study the Hardy type inequalities in the framework of equalities. We present equalities which imply Hardy type inequalities by dropping remainders. A characterization of functions is given on vanishing remainders. This provides a simple and direct understanding of the Hardy type inequalities as well as the nonexistence of non-trivial extremizers.

Yutaka Terasawa (Nagoya University, Japan)

A remark on Liouville-type theorems for the stationary Navier-Stokes equations in three space dimensions

Abstract

We consider the 3D homogeneous stationary Navier-Stokes equations in the whole space $\mathbb{R}^3$. We deal with solutions vanishing at infinity in the class of the finite Dirichlet integral. By means of quantities having the same scaling property as the Dirichlet integral, we establish new a priori estimates. As an application, we prove Liouville-type theorems in the marginal case of scaling invariance. This talk is based on a joint work with Hideo Kozono (Waseda Univ.) and Yuta Wakasugi (Nagoya Univ.).

Michael Ruzhansky (Imperial College London, U.K.)

Fourier multipliers on groups

Abstract

In this talk we will review the recent progress on the works on Fourier multipliers on groups of different types. Our emphasis is on Fourier multipliers as opposed to spectral multipliers: in the case of Fourier multipliers much less is known - to our knowledge, the only known cases (apart from the Euclidean space) are those of the torus, and of SU(2) by Coifman and Weiss.

We will present Hörmander-Mikhlin type L^p-multiplier theorems on general compact Lie groups (thus including torus, SU(2), 3-sphere, groups of matrices), and on graded Lie groups (thus including the Heisenberg group, Carnot groups, etc). We also present $L^p - L^q$ Fourier multiplier theorems on compact homogeneous manifolds (e.g. compact Lie groups, spheres, projective spaces, symmetric spaces), and on general unimodular locally compact groups (thus including all the above groups plus discrete groups, etc.). In the latter case it is already an interesting question what is a Fourier multiplier (e.g. if there is no Fourier analysis), but we can still handle it using appropriate constructions from von Neumann algebras and generalizations of Hardy-Littlewood and Hausdorff-Young-Paley inequalities.

The talk is based on joint works with J. Wirth (Stuttgart), V. Fischer (Bath) and R. Akylzhanov (Imperial) on different parts of this research.