ABSTRACTS

Harmonic Analysis and its Applications in Matsumoto 2016, summer

24th - 28th, August 2016

at

Shinshu University (Matsumoto Campus)

[Lecture A]

Xavier Tolsa

(ICREA / Universitat Autònoma de Barcelona, Spain)

The Riesz transform, rectifiabilty, and harmonic measure

Abstract

The first two lectures will be devoted to the so called David-Semmes problem and other related results. Roughly speaking, the David-Semmes problem consists in proving that, given a set $E \subset \mathbb{R}^d$ with finite Hausdorff n-dimensional measure H^n, the L^2 boundedness of the n-dimensional Riesz transform with respect to the measure $H^n|_E$ implies the n-rectifiability of E. In the late 1990's, in the case $n = 1$ this problem was solved for AD-regular sets by Mattila, Melnikov and Verdera, and in full generality by David and Léger, by using the connection between Menger curvature and the Cauchy kernel. The case of codimension 1 ($n = d - 1$) was solved more recently by Nazarov, Tolsa and Volberg, by combining quasiorthogonality and variational arguments. The David-Semmes problem is still open for n different from 1 and $d-1$.

The third lecture of this series will be devoted to some applications of the David-Semmes problem to the study of the connection between harmonic measure and rectifiability. In particular, I will explain a recent result by Azzam, Hofmann, Martell, Mayboroda, Mourgoglou, Tolsa and Volberg which asserts that if the harmonic measure is absolutely continuous with respect to the Hausdorff measure H^n on some subset F of the boundary of an open set in $\mathbb{R}^{n+1}$, with $H^n(F) < \infty$, then F is n-rectifiable. This result can be considered as a converse of the famous theorem of the Riesz brothers on harmonic measure for simply connected domains in the plane, with no topological assumptions in $\mathbb{R}^{n+1}$. Further, I will also describe another recent application to a two phase problem for harmonic measure by Azzam, Mourgoglou and Tolsa.

[Lecture B]

Nguyen Cong Phuc

(Louisiana State University, U.S.)

Pointwise and weighted Calderón-Zygmund type estimates with applications to nonlinear PDEs

Abstract

We discuss recent advances in pointwise potential estimates and weighted Calderón-Zygmund type estimates for quasilinear elliptic equations with measure or distributional data. The connection of those estimates to Sobolev capacities and trace inequalities as well as other intermediate estimates is presented. As an application, we discuss some results on existence criteria, removable singularities, and estimates on the size of singular sets for nonlinear equations with super-critical non-linearities such as

$$-\Delta u = u^q + \sigma \quad \text{or} - \Delta u = |\nabla u|^q + \sigma$$

(σ being a measure or a distribution) and their generalizations. A related result on the stationary Navier-Stokes equations with strongly singular external forces will also be presented.

[Lecture C]

Árpád Bényi

(Western Washington University, U.S.)

Bilinear operators, commutators and smoothing

Abstract

We will begin by reviewing some well-known facts about linear Calderón-Zygmund operators and Muckenhoupt weights, as well as the classical results of Coifman-Rochberg-Weiss and Uchiyama on the L^p-boundedness, respectively compactness, of the commutator of the Hilbert transform with BMO, respectively CMO, functions. We will then move on to the bilinear setting and discuss the smoothing effect of the commutators of certain classes of bilinear operators and functions in appropriate spaces. For example, for (p, q, r) a Hölder triple, we will prove the weighted $L^p(w_1) \times L^q(w_2)$ to $L^r(w_3)$ compactness of commutators of bilinear Calderón-Zygmund operators with CMO functions (for appropriate weights $w_j, j = 1, 2, 3$), as well as the $L^p \times L^q$ to L^r boundedness of commutators of certain classes of bilinear pseudo-differential operators that fall outside the bilinear Calderón-Zygmund theory with Lipschitz functions.

The talks will be based on several joint works with W. Damían (University of Helsinki), K. Moen (University of Alabama), R.H. Torres (University of Kansas) and V. Naibo (Kansas State University).

[Lecture D]

Keith Rogers

(ICMAT-CSIC, Spain)

Pointwise convergence to initial data

Abstract

As time goes back to zero, classical solutions of PDE converge pointwise to their initial data. When the PDE is interpreted for more general classes of initial data, this property no longer necessarily holds. Instead one may hope for the convergence to occur at almost every point or, even better, on every point off a lower dimensional set. We will begin with the simple analysis of the heat equation, and consider how regular the data must be to guarantee

1) pointwise convergence,

2) almost everywhere convergence,

and the intermediate

3) convergence off fractal sets.

We will then pass to the more difficult Schrödinger and wave equations, and consider the same three convergence properties - this encompasses an outstanding question of Carleson from 1979. A number of important tools from harmonic analysis and geometric measure theory will be introduced, including the average decay rates of the Fourier transform of fractal measures.

[Talk A]

Tomoya Kato

(Nagoya University, Japan)

Well-posedness for the generalized Zakharov-Kuznetsov equation on modulation spaces

Abstract

In this talk, we will consider the small data global well-posedness for the generalized 2D Zakharov-Kuznetzov equation in modulation spaces $M^s_{p,q}$. In order to obtain the well-posedness, some linear estimates are needed. We mainly focus on one of them called the maximal function estimate. If we re-establish this estimate in the frame of modulation spaces, the required regularity becomes lower than that in the Sobolev spaces. As a results, we obtain the global well-posedness with small initial data in $M^0_{2,1}$, which is the well-posedness in a new class of functions which is not treated by that in the Sobolev spaces.

[Talk B]

Masaharu Kobayashi

(Hokkaido University, Japan)

Operating functions on modulation and Wiener amalgam spaces

Abstract

In this talk, we characterize the operating functions on modulation spaces $M^{p,1}(\mathbb{R})$ and Wiener amalgam spaces $W^{p,1}(\mathbb{R})$. This characterization gives an affirmative answer to the open problem proposed by Bhimani and Ratnakumar.

This is joint work with Professor Enji Sato (Yamagata University).

[Talk C]

Yoshihiro Sawano

(Tokyo Metropolitan University, Japan)

Morrey spaces–Interpolations and weighted theory

Abstract

Morrey spaces are equipped with two parameters p and q, p seems to describe the global integrability of functions and q seems to describe the local integrability of functions. So, we expect Morrey spaces to grasp integrability of functions more precisely than Lebesgue spaces. However, due to the "sup" in the definition, there are many problem. First of all, Morrey spaces are not reflexive, the set of all compactly supported functions does not form a dense subset. Besides these bad properties, we do not know so much about the interpolation and its weighted theory. In this talk, we address the problem of complex interpolation and the boundedness of the Hardy-Littlewood maximal operators.

This is a joint work with Dr. Denny Ivanal Hakim, Dr. Shohei Nakamura and Professor Hitoshi Tanaka.

[Talk D]

Ryo Takada

(Tohoku University, Japan)

Dispersive estimates for the stably stratified Boussinesq equations

Abstract

We consider the initial value problem for the 3D Boussinesq equations for stably stratified fluids without the rotational effect:

$$\begin{cases} \partial_t u + (u \cdot \nabla) u = \Delta u - \nabla p + \theta e_3 & t > 0,\ x \in \mathbb{R}^3, \\ \partial_t \theta + (u \cdot \nabla)\theta = \Delta \theta - N^2 u_3 & t > 0,\ x \in \mathbb{R}^3, \\ \nabla \cdot u = 0 & t > 0,\ x \in \mathbb{R}^3, \\ u(0,x) = u_0(x), \quad \theta(0,x) = \theta_0(x) & x \in \mathbb{R}^3. \end{cases}$$

The unknown functions $u = (u_1(t,x), u_2(t,x), u_3(t,x))^T$, $p = p(t,x)$ and $\theta = \theta(t,x)$ represent the velocity field, the pressure and the thermal disturbance of the fluids, respectively, and $N > 0$ is the buoyancy frequency. The above system exhibits a dispersive nature due to the presence of the stable stratification $-N^2 u_3$. This phenomenon is closely related to the dispersive estimates for the operator $e^{\pm iNt\frac{|D_h|}{|D|}}$ defined by

$$e^{\pm iNt\frac{|D_h|}{|D|}} f(x) = \frac{1}{(2\pi)^3} \int_{\mathbb{R}^3} e^{ix\cdot\xi \pm iNt\frac{|\xi_h|}{|\xi|}} \widehat{f}(\xi)\, d\xi, \quad (t,x) \in \mathbb{R}^{1+3}.$$

We establish the sharp dispersive estimate for the linear propagator $e^{\pm iNt\frac{|D_h|}{|D|}}$. As an application, we give the explicit relation between the size of initial data and the buoyancy frequency which ensures the unique existence of global solutions to our system. In particular, it is shown that the size of the initial thermal disturbance can be taken large in proportion to the strength of stratification.

This talk is based on the joint work with Sanghyuk Lee (Seoul).

[Talk E]

Hidemitsu Wadade

(Kanazawa Universirty, Japan)

Remarks on the Hardy and Rellich type inequalities

Abstract

In this talk, we give some remarks on Hardy and Rellich type inequalities. It is well-known that the classical Hardy inequality does not admit an extremizer attaining its best constant. This fact implies that there are possibilities to improve the Hardy inequality by adding the remainder terms, and in fact we can find many papers on this direction. However, in this talk, we re-consider the classical proof of Hardy inequalities and establish not Hardy inequalities but Hardy equalities. This equalities directly imply non-existence of exremizers for the Hardy inequalities. We also consider the corresponding equalities for the Rellich inequality which is a weighted Sobolev type inequality involving the second order derivatives.

These results are joint works with Professor Tohru Ozawa from Waseda University and Professor Shuji Machihara from Saitama University.