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Definition of M_G

Let G be a finite subgroup of $\mathrm{GL}(n, \mathbb{Z})$. The G -lattice M_G of rank n is defined to be the G -lattice with a \mathbb{Z} -basis $\{u_1, \dots, u_n\}$ on which G acts by $\sigma(u_i) = \sum_{j=1}^n a_{i,j} u_j$ for any $\sigma = [a_{i,j}] \in G$.

Hminus1

▸ `Hminus1(G)`

returns the Tate cohomology group $\widehat{H}^{-1}(G, M_G)$ for a finite subgroup $G \leq \mathrm{GL}(n, \mathbb{Z})$.

H0

▸ `H0(G)`

returns the Tate cohomology group $\widehat{H}^0(G, M_G)$ for a finite subgroup $G \leq \mathrm{GL}(n, \mathbb{Z})$.

H1

▸ `H1(G)`

returns the cohomology group $H^1(G, M_G)$ for a finite subgroup $G \leq \mathrm{GL}(n, \mathbb{Z})$.

Z0lattice

▸ `Z0lattice(G)`

returns a \mathbb{Z} -basis of the group of Tate 0-cocycles $\widehat{Z}^0(G, M_G)$ for a finite subgroup $G \leq \mathrm{GL}(n, \mathbb{Z})$.

ConjugacyClassesSubgroups2, ConjugacyClassesSubgroupsFromPerm

▸ `ConjugacyClassesSubgroups2(G)`

▸ `ConjugacyClassesSubgroupsFromPerm(G)`

returns the list of conjugacy classes of subgroups of a group G . We use this function because the ordering of the conjugacy classes of subgroups of G by the built-in function `ConjugacyClassesSubgroups(G)` is not fixed for some groups. If a group G is too big, `ConjugacyClassesSubgroups2(G)` may not work well.

IsFlabby

```
▸ IsFlabby( $G$ )
```

returns whether G -lattice M_G is flabby or not.

IsCoflabby

```
▸ IsCoflabby( $G$ )
```

returns whether G -lattice M_G is coflabby or not.

IsInvertible

```
▸ IsInvertible( $G$ )
```

returns whether G -lattice M_G is invertible or not.

FlabbyResoluton

```
▸ FlabbyResolution( $G$ )
```

returns a flabby resolution $0 \rightarrow M_G \xrightarrow{\iota} P \xrightarrow{\phi} F \rightarrow 0$ of M_G as follows:

```
▸ FlabbyResolution( $G$ ).actionP
```

returns the matrix representation of the action of G on P ;

```
▸ FlabbyResolution( $G$ ).actionF
```

returns the matrix representation of the action of G on F ;

```
▸ FlabbyResolution( $G$ ).injection
```

returns the matrix which corresponds to the injection $\iota : M_G \rightarrow P$;

```
▸ FlabbyResolution( $G$ ).surjection
```

returns the matrix which corresponds to the surjection $\phi : P \rightarrow F$.

IsInvertibleF

```
▸ IsInvertibleF( $G$ )
```

returns whether $[M_G]^{fl}$ is invertible.

flfl

▸ flfl(G)

returns the G -lattice E with $[[M_G]^{fl}]^{fl} = [E]$.

PossibilityOfStablyPermutationF

▸ PossibilityOfStablyPermutationF(G)

returns a basis $\mathcal{L} = \{l_1, \dots, l_s\}$ of the solution space of the system of linear equations which is obtained by computing some \mathbb{Z} -class invariants. Each isomorphism class of irreducible permutation G -lattices corresponds to a conjugacy class of subgroup H of G by $H \leftrightarrow \mathbb{Z}[G/H]$. Let H_1, \dots, H_r be conjugacy classes of subgroups of G whose ordering corresponds to the GAP function `ConjugacyClassesSubgroups2(G)`. Let F be the flabby class of M_G . We assume that F is stably permutation, i.e. for $x_{r+1} = \pm 1$,

$$\left(\bigoplus_{i=1}^r \mathbb{Z}[G/H_i]^{\oplus x_i} \right) \oplus F^{\oplus x_{r+1}} \simeq \bigoplus_{i=1}^r \mathbb{Z}[G/H_i]^{\oplus y_i}.$$

Define $a_i = x_i - y_i$ and $b_1 = x_{r+1}$. Then we have for $b_1 = \pm 1$,

$$\bigoplus_{i=1}^r \mathbb{Z}[G/H_i]^{\oplus a_i} \simeq F^{\oplus (-b_1)}.$$

$[M_G]^{fl} = 0 \implies$ there exist $a_1, \dots, a_r \in \mathbb{Z}$ and $b_1 = \pm 1$ which satisfy the system of linear equations.

PossibilityOfStablyPermutationM

▸ PossibilityOfStablyPermutationM(G)

returns the same as `PossibilityOfStablyPermutationF(G)` but with respect to M_G instead of F .

Nlist

▸ Nlist(l)

returns the negative part of the list l .

Plist

▸ Plist(l)

returns the positive part of the list l .

StablyPermutationFCheck

▸ `StablyPermutationFCheck($G, L1, L2$)`

returns the matrix P which satisfies $G_1 P = P G_2$ where G_1 (resp. G_2) is the matrix representation group of the action of G on $(\bigoplus_{i=1}^r \mathbb{Z}[G/H_i]^{\oplus a_i}) \oplus F^{\oplus b_1}$ (resp. $(\bigoplus_{i=1}^r \mathbb{Z}[G/H_i]^{\oplus a'_i}) \oplus F^{\oplus b'_1}$) with the isomorphism

$$\left(\bigoplus_{i=1}^r \mathbb{Z}[G/H_i]^{\oplus a_i} \right) \oplus F^{\oplus b_1} \simeq \left(\bigoplus_{i=1}^r \mathbb{Z}[G/H_i]^{\oplus a'_i} \right) \oplus F^{\oplus b'_1}$$

for lists $L_1 = [a_1, \dots, a_r, b_1]$ and $L_2 = [a'_1, \dots, a'_r, b'_1]$, if P exists. If such P does not exist, this returns false.

StablyPermutationMCheck

▸ `StablyPermutationMCheck($G, L1, L2$)`

returns the same as `StablyPermutationFCheck($G, L1, L2$)` but with respect to M_G instead of F .

StablyPermutationFCheckP

▸ `StablyPermutationFCheckP($G, L1, L2$)`

returns a basis $\mathcal{P} = \{P_1, \dots, P_m\}$ of the solution space of $G_1 P = P G_2$ where G_1 (resp. G_2) is the matrix representation group of the action of G on $(\bigoplus_{i=1}^r \mathbb{Z}[G/H_i]^{\oplus a_i}) \oplus F^{\oplus b_1}$ (resp. $(\bigoplus_{i=1}^r \mathbb{Z}[G/H_i]^{\oplus a'_i}) \oplus F^{\oplus b'_1}$) for lists $L_1 = [a_1, \dots, a_r, b_1]$ and $L_2 = [a'_1, \dots, a'_r, b'_1]$, if P exists. If such P does not exist, this returns `[]`.

StablyPermutationMCheckP

▸ `StablyPermutationMCheckP($G, L1, L2$)`

returns the same as `StablyPermutationFCheckP($G, L1, L2$)` but with respect to M_G instead of F .

StablyPermutationFCheckMat

▸ `StablyPermutationFCheckMat($G, L1, L2, P$)`

returns true if $G_1 P = P G_2$ and $\det P = \pm 1$ where G_1 (resp. G_2) is the matrix representation group of the action of G on $(\bigoplus_{i=1}^r \mathbb{Z}[G/H_i]^{\oplus a_i}) \oplus F^{\oplus b_1}$ (resp. $(\bigoplus_{i=1}^r \mathbb{Z}[G/H_i]^{\oplus a'_i}) \oplus F^{\oplus b'_1}$) for lists $L_1 = [a_1, \dots, a_r, b_1]$ and $L_2 = [a'_1, \dots, a'_r, b'_1]$. If not, this returns false.

StablyPermutationMCheckMat

▸ `StablyPermutationMCheckMat($G, L1, L2, P$)`

returns the same as `StablyPermutationFCheckMat($G, L1, L2, P$)` but with respect to M_G instead of F .

StablyPermutationFCheckGen

▸ `StablyPermutationFCheckP($G, L1, L2$)`

returns the list $[\mathcal{M}_1, \mathcal{M}_2]$ where $\mathcal{M}_1 = [g_1, \dots, g_t]$ (resp. $\mathcal{M}_2 = [g'_1, \dots, g'_t]$) is a list of the generators of G_1 (resp. G_2) which is the matrix representation group of the action of G on $(\bigoplus_{i=1}^r \mathbb{Z}[G/H_i]^{\oplus a_i}) \oplus F^{\oplus b_1}$ (resp. $(\bigoplus_{i=1}^r \mathbb{Z}[G/H_i]^{\oplus a'_i}) \oplus F^{\oplus b'_1}$) for lists $L_1 = [a_1, \dots, a_r, b_1]$ and $L_2 = [a'_1, \dots, a'_r, b'_1]$.

StablyPermutationMCheckGen

▸ `StablyPermutationMCheckGen($G, L1, L2$)`

returns the same as `StablyPermutationFCheckGen($G, L1, L2$)` but with respect to M_G instead of F .

DirectSumMatrixGroup

▸ `DirectSumMatrixGroup(L)`

returns the direct sum of the groups G_1, \dots, G_n for the list $l = [G_1, \dots, G_n]$.

DirectProductMatrixGroup

▸ `DirectProductMatrixGroup(L)`

returns the direct product of the groups G_1, \dots, G_n for the list $l = [G_1, \dots, G_n]$.

Norm1TorusJ

▸ `Norm1TorusJ(d, m)`

returns the Chevalley module $J_{G/H}$ for the m -th transitive subgroup $G = dTm \leq S_d$ of degree d where H is the stabilizer of one of the letters in G .

FlabbyResolutionLowRankFromGroup

▸ `FlabbyResolutionLowRankFromGroup(M, G)`

returns a suitable flabby resolution with low rank for G -lattice M by using

backtracking techniques. Repeating the algorithm, by defining $[M]^{f^n} := [[M]^{f^{n-1}}]^{f^l}$ inductively, $[M]^{f^l} = 0$ is provided if we may find some n with $[M]^{f^n} = 0$ (this method is slightly improved to the flfl algorithm, see above).

Hcandidates

▸ Hcandidates(G)

retruns subgroups H of G which satisfy $\bigcap_{\sigma \in G} H^\sigma = \{1\}$ where $H^\sigma = \sigma^{-1}H\sigma$ (hence H contains no normal subgroup of G except for $\{1\}$).

Norm1TorusJTransitiveGroup

▸ Norm1TorusJTransitiveGroup(d, m)

returns the Chevalley module $J_{G/H}$ for the m -th transitive subgroup $G = {}_dT_m \leq S_d$ of degree d where H is the stabilizer of one of the letters in G . (The input and output of this function is the same as the function Norm1TorusJ(d, m) but this function is more efficient.)

Norm1TorusJCoset

▸ Norm1TorusJCoset(G, H)

retruns the Chevalley module $J_{G/H}$ for a group G and a subgroup $H \leq G$.

StablyPermutationMCheckPPari

▸ StablyPermutationMCheckPPari($G, L1, L2$)

returns the same as StablyPermutationMCheckP($G, L1, L2$) but using efficient PARI/GP functions (e.g. matker, matsnf) [PARI2]. (This function applies union-find algorithm and it also requires PARI/GP [PARI2].)

StablyPermutationFCheckPPari

▸ StablyPermutationFCheckPPari($G, L1, L2$)

returns the same as StablyPermutationFCheckP($G, L1, L2$) but using efficient PARI/GP functions (e.g. matker, matsnf) [PARI2]. (This function applies union-find algorithm and it also requires PARI/GP [PARI2].)

StablyPermutationFCheckPFromBasePari

▸ StablyPermutationFCheckPFromBasePari($G, m_i, L1, L2$)

returns the same as StablyPermutationFCheckPPari($G, L1, L2$) but with respect to $m_i = \mathcal{P}^\circ$ instead of the original \mathcal{P}° as in Hoshi and Yamasaki [HY17, Equation (4) in Section 5.1]. (See [HY17, Section 5.7, Method III]. This function

applies union-find algorithm and it also requires PARI/GP [PARI2].)

References

[HY17] Akinari Hoshi and Aiichi Yamasaki, Rationality problem for algebraic tori, Mem. Amer. Math. Soc. **248** (2017) no. 1176, v+215 pp. [AMS](#) Preprint version: [arXiv:1210.4525](#).

[HHY20] Sumito Hasegawa, Akinari Hoshi and Aiichi Yamasaki, Rationality problem for norm one tori in small dimensions, Math. Comp. **89** (2020) 923-940. [AMS](#) Extended version: [arXiv:1811.02145](#).

[HY] Akinari Hoshi and Aiichi Yamasaki, Rationality problem for norm one tori for A_5 and $\mathrm{PSL}_2(\mathbb{F}_8)$ extensions, [arXiv:2309.16187](#).

[PARI2] The PARI Group, PARI/GP version 2.13.3, Univ. Bordeaux, 2021, <http://pari.math.u-bordeaux.fr/>.

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