- Return to RatProbNorm1Tori
- Return to Norm1ToriHNP

FlabbyResolutionFromBase.gap

Definition of M_G

Let G be a finite subgroup of $\mathrm{GL}(n,\mathbb{Z})$. The G-lattice M_G of rank n is defined to be the G-lattice with a $\mathbb{Z}$-basis $\{u_1,\ldots,u_n\}$ on which G acts by $\sigma(u_i)=\sum_{j=1}^n a_{i,j}u_j$ for any $\sigma=[a_{i,j}]\in G$.

Hminus1

```
‣ Hminus1(G)
```

returns the Tate cohomology group $\widehat{H}^{-1}(G,M_G)$ for a finite subgroup $G\leq \mathrm{GL}(n,\mathbb{Z})$.

H0

```
‣ H0(G)
```

returns the Tate cohomology group $\widehat{H}^{0}(G,M_G)$ for a finite subgroup $G\leq \mathrm{GL}(n,\mathbb{Z})$.

H1

```
‣ H1(G)
```

returns the cohomology group $H^1(G,M_G)$ for a finite subgroup $G\leq \mathrm{GL}(n,\mathbb{Z})$.

Z0lattice

```
‣ Z0lattice(G)
```

returns a $\mathbb{Z}$-basis of the group of Tate 0-cocycles $\widehat{Z}^0(G,M_G)$ for a finite subgroup $G\leq \mathrm{GL}(n,\mathbb{Z})$.

ConjugacyClassesSubgroups2, ConjugacyClassesSubgroupsFromPerm

```
‣ ConjugacyClassesSubgroups2(G)
```

```
‣ ConjugacyClassesSubgroupsFromPerm(G)
```

returns the list of conjugacy classes of subgroups of a group G. We use this function because the ordering of the conjugacy classes of subgroups of G by the built-in function ConjugacyClassesSubgroups(*G*) is not fixed for some groups. If a group G is too big, ConjugacyClassesSubgroups2(*G*) may not work well.

IsFlabby

```
‣ IsFlabby(G)
```

returns whether G-lattice M_G is flabby or not.

IsCoflabby

```
‣ IsCoflabby(G)
```

returns whether G-lattice M_G is coflabby or not.

IsInvertible

```
‣ IsInvertible(G)
```

returns whether G-lattice M_G is invertible or not.

FlabbyResoluton

```
‣ FlabbyResolution(G)
```

returns a flabby resolution $0 \to M_G \xrightarrow{\iota} P \xrightarrow{\phi} F \to 0$ of M_G as follows:

```
‣ FlabbyResolution(G).actionP
```

returns the matrix representation of the action of G on P;

```
‣ FlabbyResolution(G).actionF
```

returns the matrix representation of the action of G on F;

```
‣ FlabbyResolution(G).injection
```

returns the matrix which corresponds to the injection $\iota : M_G \to P$;

```
‣ FlabbyResolution(G).surjection
```

returns the matrix which corresponds to the surjection $\phi : P \to F$.

IsInvertibleF

```
‣ IsInvertibleF(G)
```

returns whether $[M_G]^{fl}$ is invertible.

flfl

```
‣ flfl(G)
```

returns the G-lattice E with $[[M_G]^{fl}]^{fl} = [E]$.

PossibilityOfStablyPermutationF

```
‣ PossibilityOfStablyPermutationF(G)
```

returns a basis $\mathcal{L} = \{l_1, \ldots, l_s\}$ of the solution space of the system of linear equations which is obtained by computing some $\mathbb{Z}$-class invariants. Each isomorphism class of irreducible permutation G-lattices corresponds to a conjugacy class of subgroup H of G by $H \leftrightarrow \mathbb{Z}[G/H]$. Let $H_1, \ldots, H_r$ be conjugacy classes of subgroups of G whose ordering corresponds to the GAP function ConjugacyClassesSubgroups2(*G*). Let F be the flabby class of M_G. We assume that F is stably permutation, i.e. for $x_{r+1} = \pm 1$,

$$\left(\bigoplus_{i=1}^{r} \mathbb{Z}[G/H_i]^{\oplus x_i}\right) \oplus F^{\oplus x_{r+1}} \ \simeq\ \bigoplus_{i=1}^{r} \mathbb{Z}[G/H_i]^{\oplus y_i}.$$

Define $a_i = x_i - y_i$ and $b_1 = x_{r+1}$. Then we have for $b_1 = \pm 1$,

$$\bigoplus_{i=1}^{r} \mathbb{Z}[G/H_i]^{\oplus a_i} \ \simeq\ F^{\oplus(-b_1)}.$$

$[M_G]^{fl} = 0 \Longrightarrow$ there exist $a_1, \ldots, a_r \in \mathbb{Z}$ and $b_1 = \pm 1$ which satisfy the system of linear equations.

PossibilityOfStablyPermutationM

```
‣ PossibilityOfStablyPermutationM(G)
```

returns the same as PossibilityOfStablyPermutationF(*G*) but with respect to M_G instead of F.

Nlist

```
‣ Nlist(L)
```

returns the negative part of the list l.

Plist

```
‣ Plist(L)
```

returns the positive part of the list l.

StablyPermutationFCheck

```
‣ StablyPermutationFCheck(G,L1,L2)
```

returns the matrix P which satisfies $G_1P = PG_2$ where G_1 (resp. G_2) is the matrix representation group of the action of G on $(\oplus_{i=1}^{r}\mathbb{Z}[G/H_i]^{\oplus a_i}) \oplus F^{\oplus b_1}$ (resp. $(\oplus_{i=1}^{r}\mathbb{Z}[G/H_i]^{\oplus a_i'}) \oplus F^{\oplus b_1'}$) with the isomorphism

$$\left(\bigoplus_{i=1}^{r}\mathbb{Z}[G/H_i]^{\oplus a_i}\right) \oplus F^{\oplus b_1} \simeq \left(\bigoplus_{i=1}^{r}\mathbb{Z}[G/H_i]^{\oplus a_i'}\right) \oplus F^{\oplus b_1'}$$

for lists $L_1 = [a_1, \ldots, a_r, b_1]$ and $L_2 = [a_1', \ldots, a_r', b_1']$, if P exists. If such P does not exist, this returns false.

StablyPermutationMCheck

```
‣ StablyPermutationMCheck(G,L1,L2)
```

returns the same as StablyPermutationFCheck(G,L1,L2) but with respect to M_G instead of F.

StablyPermutationFCheckP

```
‣ StablyPermutationFCheckP(G,L1,L2)
```

returns a basis $\mathcal{P} = \{P_1, \ldots, P_m\}$ of the solution space of $G_1P = PG_2$ where G_1 (resp. G_2) is the matrix representation group of the action of G on $(\oplus_{i=1}^{r}\mathbb{Z}[G/H_i]^{\oplus a_i}) \oplus F^{\oplus b_1}$ (resp. $(\oplus_{i=1}^{r}\mathbb{Z}[G/H_i]^{\oplus a_i'}) \oplus F^{\oplus b_1'}$) for lists $L_1 = [a_1, \ldots, a_r, b_1]$ and $L_2 = [a_1', \ldots, a_r', b_1']$, if P exists. If such P does not exist, this returns [ ].

StablyPermutationMCheckP

```
‣ StablyPermutationMCheckP(G,L1,L2)
```

returns the same as StablyPermutationFCheckP(G,L1,L2) but with respect to M_G instead of F.

StablyPermutationFCheckMat

```
‣ StablyPermutationFCheckMat(G,L1,L2,P)
```

returns true if $G_1P = PG_2$ and det $P = \pm 1$ where G_1 (resp. G_2) is the matrix representation group of the action of G on $(\oplus_{i=1}^{r}\mathbb{Z}[G/H_i]^{\oplus a_i}) \oplus F^{\oplus b_1}$ (resp. $(\oplus_{i=1}^{r}\mathbb{Z}[G/H_i]^{\oplus a_i'}) \oplus F^{\oplus b_1'}$) for lists $L_1 = [a_1, \ldots, a_r, b_1]$ and $L_2 = [a_1', \ldots, a_r', b_1']$. If not, this returns false.

StablyPermutationMCheckMat

```
‣ StablyPermutationMCheckMat(G,L1,L2,P)
```

returns the same as StablyPermutationFCheckMat(*G*,*L1*,*L2*,*P*) but with respect to M_G instead of F.

StablyPermutationFCheckGen

```
‣ StablyPermutationFCheckP(G,L1,L2)
```

returns the list $[\mathcal{M}_1,\mathcal{M}_2]$ where $\mathcal{M}_1=[g_1,\dots,g_t]$ (resp. $\mathcal{M}_2=[g_1',\dots,g_t']$) is a list of the generators of G_1 (resp. G_2) which is the matrix representation group of the action of G on $(\oplus_{i=1}^r\mathbb{Z}[G/H_i]^{\oplus a_i})\oplus F^{\oplus b_1}$ (resp. $(\oplus_{i=1}^r\mathbb{Z}[G/H_i]^{\oplus a_i'})\oplus F^{\oplus b_1'}$) for lists $L_1=[a_1,\dots,a_r,b_1]$ and $L_2=[a_1',\dots,a_r',b_1']$.

StablyPermutationMCheckGen

```
‣ StablyPermutationMCheckGen(G,L1,L2)
```

returns the same as StablyPermutationFCheckGen(*G*,*L1*,*L2*) but with respect to M_G instead of F.

DirectSumMatrixGroup

```
‣ DirectSumMatrixGroup(l)
```

returns the direct sum of the groups $G_1,\dots,G_n$ for the list $l=[G_1,\dots,G_n]$.

DirectProductMatrixGroup

```
‣ DirectProductMatrixGroup(l)
```

returns the direct product of the groups $G_1,\dots,G_n$ for the list $l=[G_1,\dots,G_n]$.

Norm1TorusJ

```
‣ Norm1TorusJ(d,m)
```

returns the Chevalley module $J_{G/H}$ for the m-th transitive subgroup $G=dTm\leq S_d$ of degree d where H is the stabilizer of one of the letters in G.

FlabbyResolutionLowRankFromGroup

```
‣ FlabbyResolutionLowRankFromGroup(M,G)
```

returns a suitable flabby resolution with low rank for G-lattice M by using backtracking techniques. Repeating the algorithm, by defining $[M]^{fl^n} := [[M]^{fl^{n-1}}]^{fl}$ inductively, $[M]^{fl} = 0$ is provided if we may find some n with $[M]^{fl^n} = 0$ (this method is slightly improved to the flfl algorithm, see above).

Hcandidates

```
‣ Hcandidates(G)
```

retruns subgroups H of G which satisfy $\bigcap_{\sigma\in G} H^\sigma = \{1\}$ where $H^\sigma = \sigma^{-1}H\sigma$ (hence H contains no normal subgroup of G except for $\{1\}$).

Norm1TorusJTransitiveGroup

```
‣ Norm1TorusJTransitiveGroup(d,m)
```

returns the Chevalley module $J_{G/H}$ for the m-th transitive subgroup $G = {}_dT_m \leq S_d$ of degree d where H is the stabilizer of one of the letters in G . (The input and output of this function is the same as the function Norm1TorusJ(*d*,*m*) but this function is more efficient.)

Norm1TorusJCoset

```
‣ Norm1TorusJCoset(G,H)
```

retruns the Chevalley module $J_{G/H}$ for a group G and a subgroup $H \leq G$.

StablyPermutationMCheckPPari

```
‣ StablyPermutationMCheckPPari(G,L1,L2)
```

returns the same as StablyPermutationMCheckP(*G*,*L1*,*L2*) but using efficient PARI/GP functions (e.g. matker, matsnf) [PARI2]. (This function applies union-find algorithm and it also requires PARI/GP [PARI2].)

StablyPermutationFCheckPPari

```
‣ StablyPermutationFCheckPPari(G,L1,L2)
```

returns the same as StablyPermutationFCheckP(*G*,*L1*,*L2*) but using efficient PARI/GP functions (e.g. matker, matsnf) [PARI2]. (This function applies union-find algorithm and it also requires PARI/GP [PARI2].)

StablyPermutationFCheckPFromBasePari

```
‣ StablyPermutationFCheckPFromBasePari(G,mi,L1,L2)
```

returns the same as StablyPermutationFCheckPPari(*G*,*L1*,*L2*) but with respect to $m_i = \mathcal{P}^\circ$ instead of the original $\mathcal{P}^\circ$ as in Hoshi and Yamasaki [HY17, Equation (4) in Section 5.1]. (See [HY17, Section 5.7, Method III]. This function applies union-find algorithm and it also requires PARI/GP [PARI2].)

FlabbyResolutionNorm1TorusJ

```
‣ FlabbyResolutionNorm1TorusJ(d,m).actionF
```

returns the matrix representation of the action of G on a flabby class $F=[J_{G/H}]^{fl}$ for the m-th transitive subgroup $G=dTm\leq S_n$ of degree n where H is the stabilizer of one of the letters in G. (This function is similar to FlabbyResolution(Norm1TorusJ(*d*,*m*)).actionF but it may speed up and save memory resources.)

References

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[HHY20] Sumito Hasegawa, Akinari Hoshi and Aiichi Yamasaki, Rationality problem for norm one tori in small dimensions, Math. Comp. **89** (2020) 923-940. AMS Extended version: arXiv:1811.02145.
[HY] Akinari Hoshi and Aiichi Yamasaki, Rationality problem for norm one tori for A_5 and $\mathrm{PSL}_2(\mathbb{F}_8)$ extensions, arXiv:2309.16187.
[PARI2] The PARI Group, PARI/GP version 2.13.3, Univ. Bordeaux, 2021, http://pari.math.u-bordeaux.fr/.