Real slice of complex surface automorphism and complex Salem number

Shigehiro Ushiki

Birational automorphism $f_{\alpha,\beta}(x,y) = (y, \frac{y+\alpha}{x+\beta} + \beta)$ has an invariant cubic curve for special values of parameters. For special cases of real parameter values of α, β, the birational map lifts to an automorphism of complex surface, leaving the real slice invariant.

THEOREM. The characteristic polynomial of the linear map of homology group induced by the real slice of surface automorphisms are :
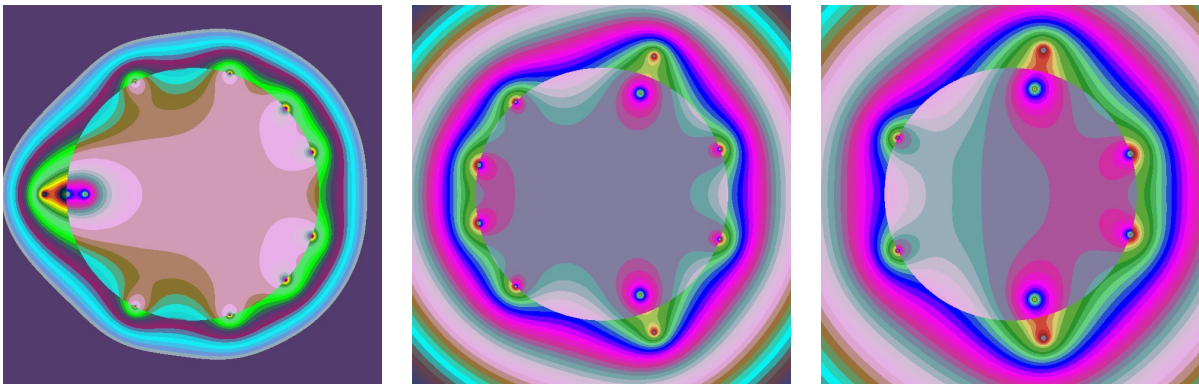
$$(\Gamma_1) \quad \phi_n(z) = \frac{1}{z+1}\{z^{n+1}(z^3 - z + 1) - (-1)^n(z^3 - z^2 + 1)\},$$

$$(\Gamma_2) \quad \psi_{2k}(z) = \frac{z+1}{z^2+1}\{z^{2k+1}(z^3 + z + 1) + (-1)^k(z^3 + z^2 + 1)\},$$

$$(\Gamma_3) \quad \varphi_{3k}(z) = \frac{z^2 + z + 1}{z^3 + 1}\{z^{3k+1}(z^3 + z + 1) - (-1)^k(z^3 + z^2 + 1)\}.$$

THEOREM. The leading eigenvalues of the homology homomorphisms are :

(Γ_1) case　　　　negative Salem number,
(Γ_2) and (Γ_3) cases　　complex Salem number.

Kyoto,2019.04.19.