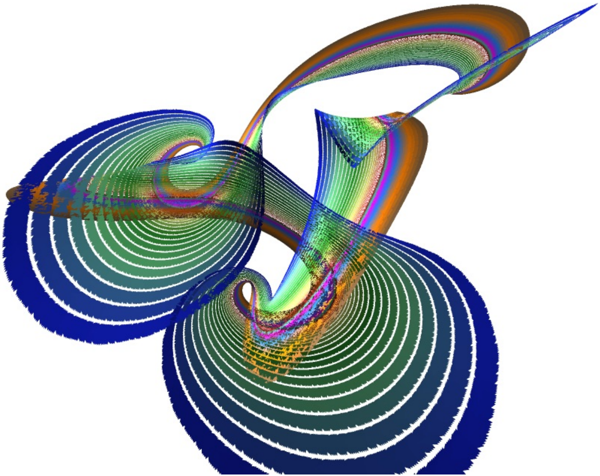
Attracting Herman rings for Hénon Dynamics



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Kyoto, Apr., 12, 2024

Abstract

Very recently, attracting Herman rings for Hénon maps are discovered by Raphael Krikorian.

Xavier Buff found that Herman rings exist for Hénon maps for real parameters.

These are first examples of Herman rings for Hénon maps with a proof (not published yet).

While waiting for their papers, let us observe the phenomena by means of computer graphics.

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1. Hénon map
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1. Hénon map

Hénon map and Fatou set

Hénon map is an automorphism $f : \mathbb{C}^2 \to \mathbb{C}^2$ defined by

$$f(x, y) = (x^2 + c + by, x).$$

A point $p \in \mathbb{C}^2$ is a point of the **forward Fatou set** F_f^+ if there exists an open neighborhood U of p on which the sequence $\{f^n\}_{n\in\mathbb{N}}$ forms a normal family of holomorphic mappings from U to X.

Dissipative Hénon map

In the Hénon map case, Bedford and Smilie [BS2] proved :

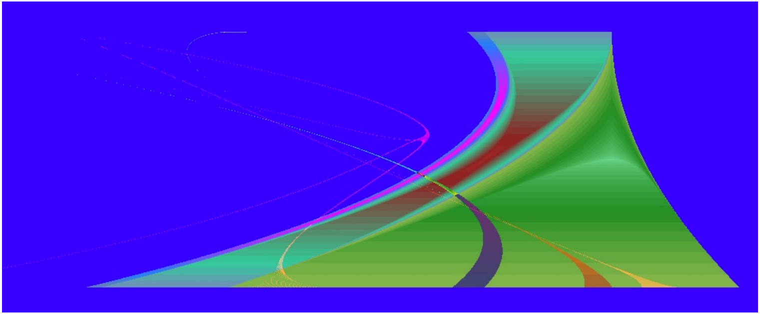
THEOREM (Bedford-Smilie, 1991) Suppose that f is dissipative, and $f(\Omega) = \Omega$ is an invariant Fatou component satisfying $\overline{\{f^n(p) : n \geq 0\}} \subset \Omega$ for some $p \in \Omega$. Then one of the following occurs:

1. $\Omega =$ basin of an attracting fixed point, and $\Omega \cong \mathbb{C}^2$.
2. $\Omega =$ basin of a rotational disk, and $\Omega \cong \mathbb{D} \times \mathbb{C}$.
3. $\Omega =$ basin of a rotational annulus, and $\Omega \cong \mathbb{A} \times \mathbb{C}$.

REM. Existence of case 3 was an open problem until fev., 2024.
REM. Existence of parabolic basins is proved by T. Ueda (1986, 1991).

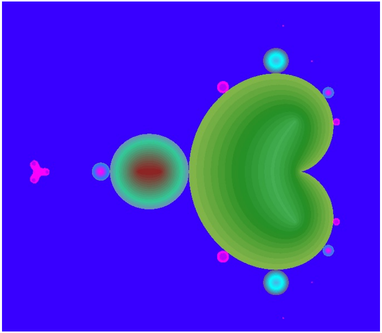
Parameter space

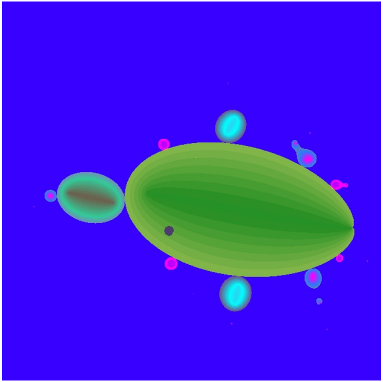
Our Hénon map has two complex parameters, $(c, b) \in \mathbb{C}^2$.



This is a real slice with coordinates (c, b).
Parameter values for which attracting periodic points (of period up to 5) exist are plotted.

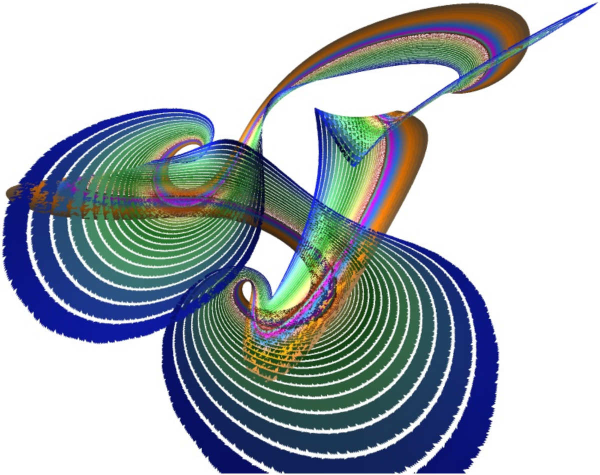
c-slice



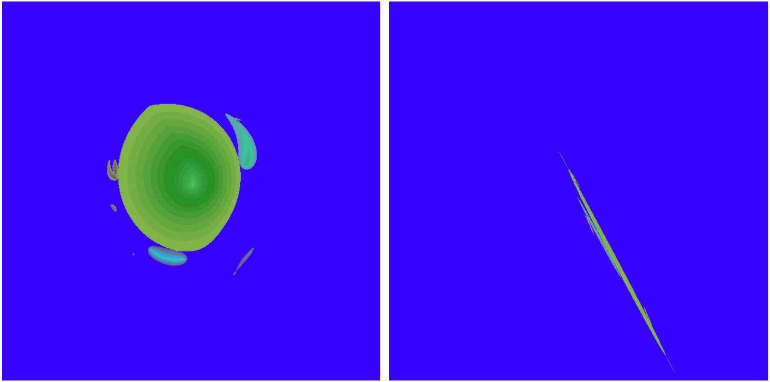


2. Krikorian's Herman ring

Krikorian's Herman ring



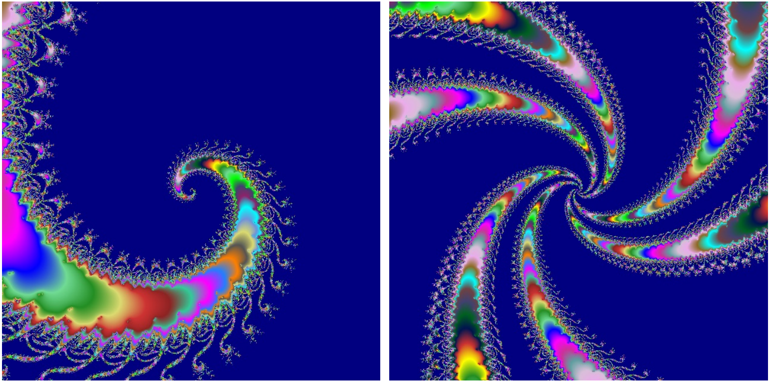
b-slice and c-slice passing the Krikorian's parameter



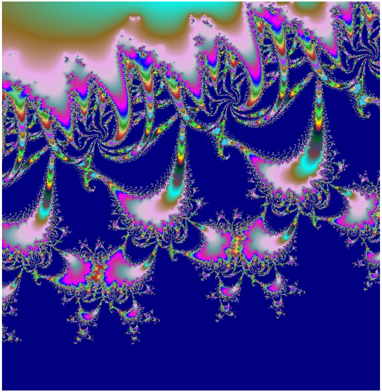
$$c = -0.1138145366 + 0.2283022835i,$$

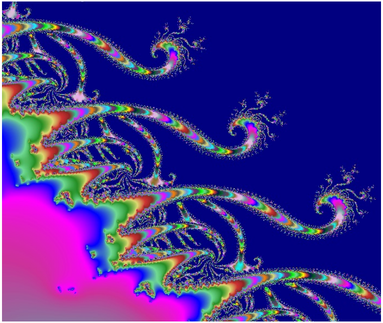
$$b = 0.4634460910 - 0.855698503i.$$

Unstable manifold of a saddle of period 3 and 1

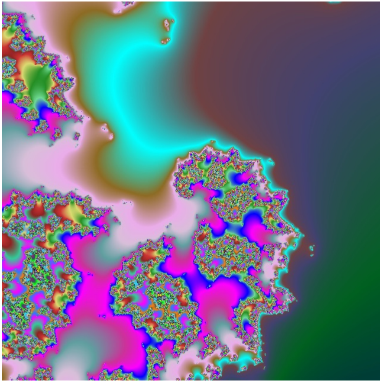
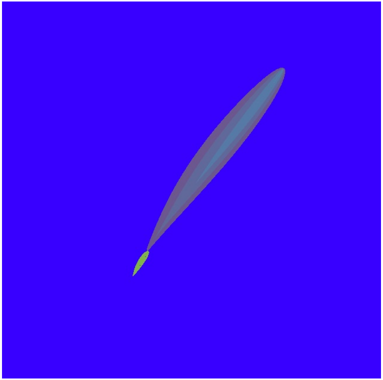


zoom im

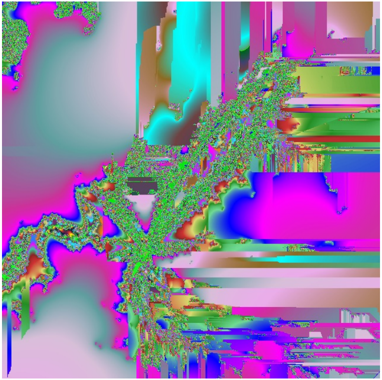


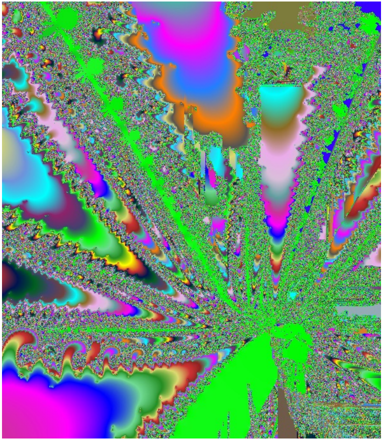


Green's function on unstable manifold

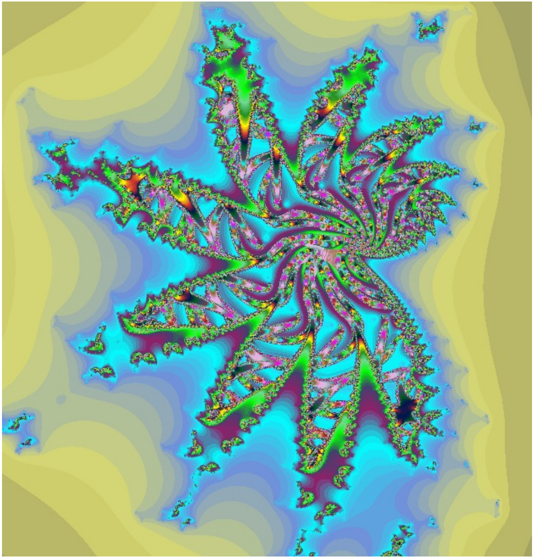
 

Saddle drop



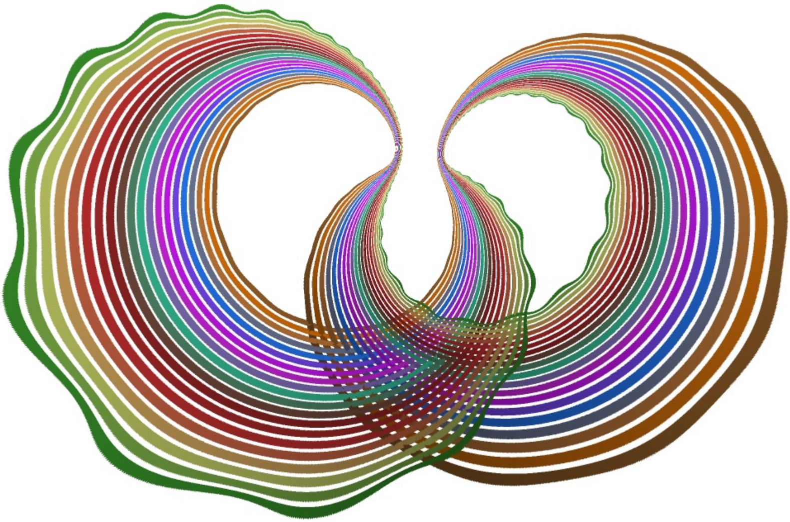


Slice

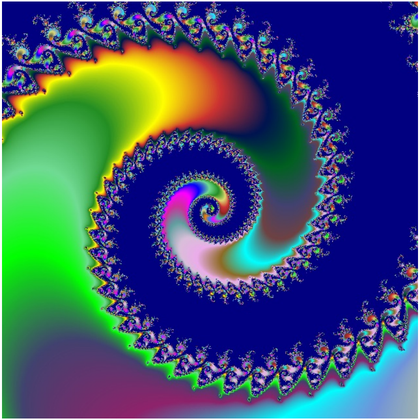


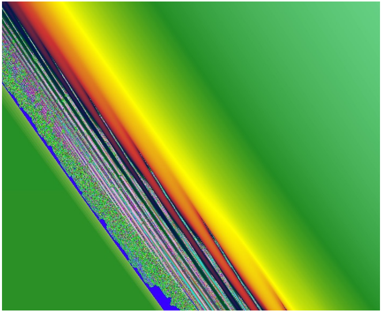
3. Buff's Herman ring

Buff's Herman ring for real parameters

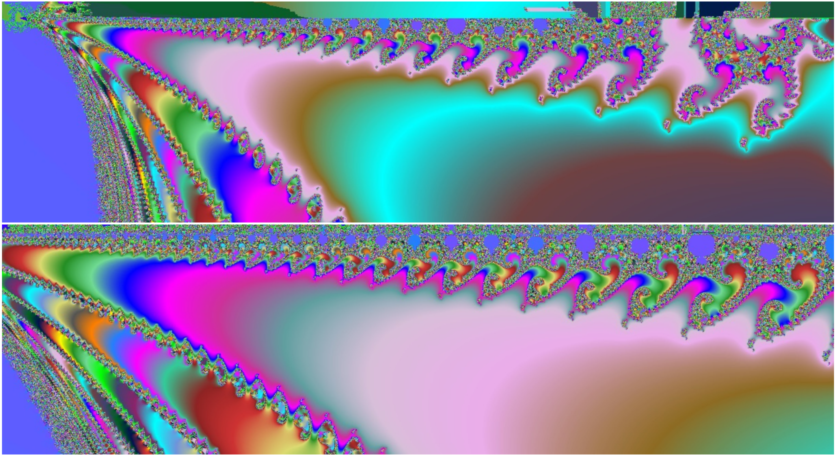


unstable manifold and real parameter space

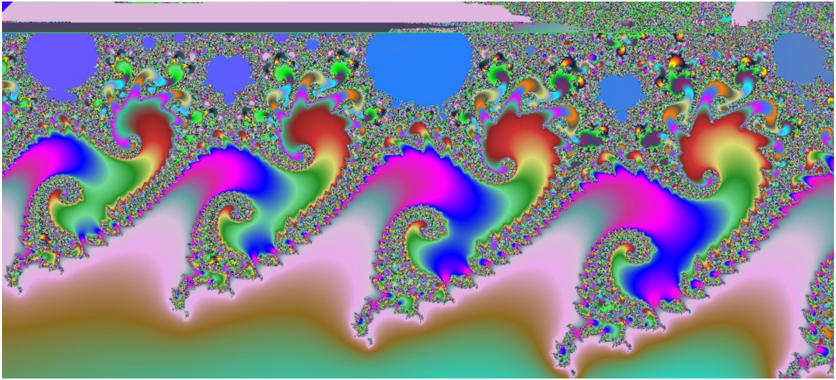




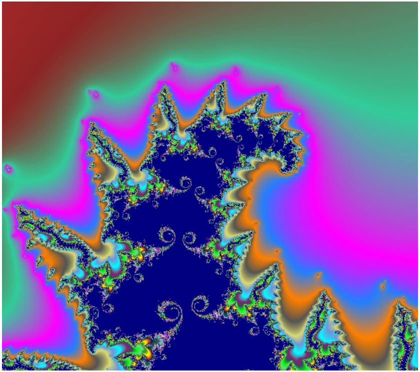
saddle drop picture (c-slice, b-slice)

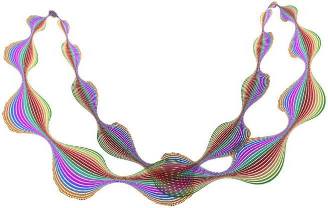


saddle drop picture (c-slice)

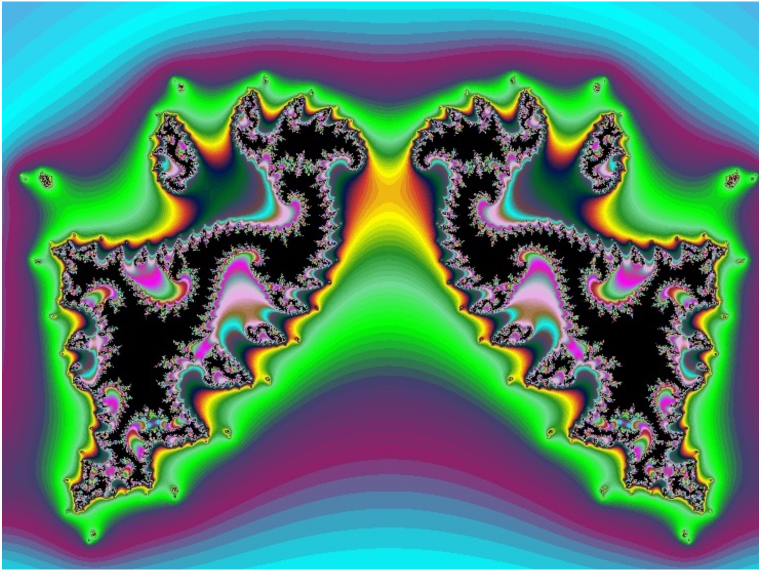


unstable manifold and Herman ring



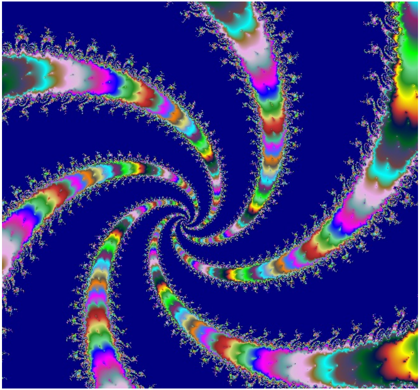


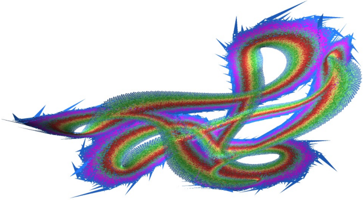
Slice



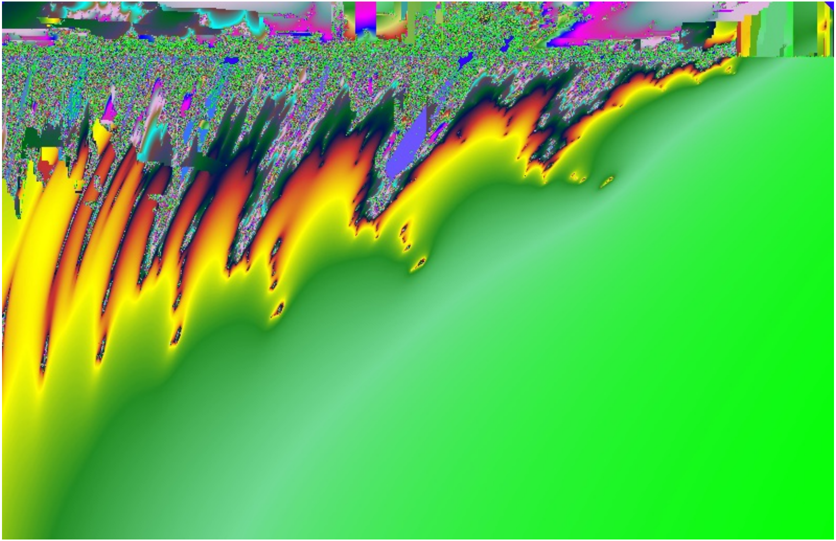
4. Herman ring for real parameters

unstable manifold and Herman ring

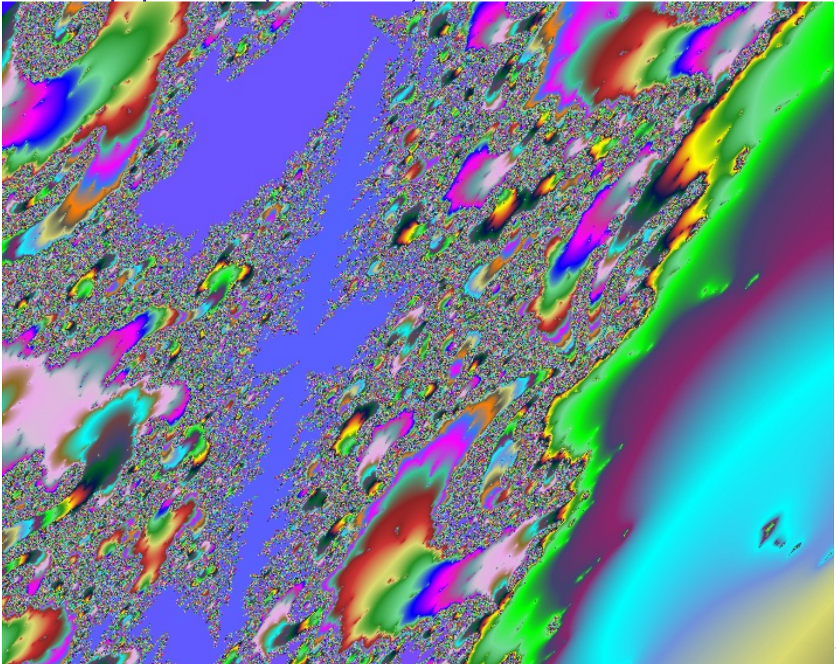




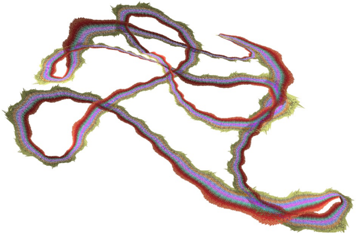
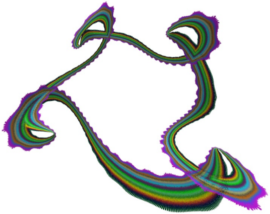
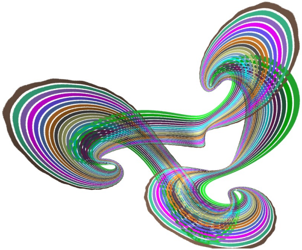
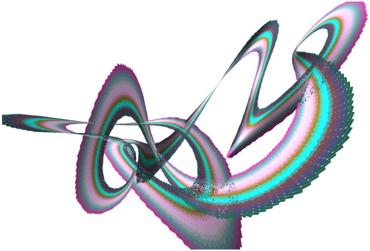
saddle drop picture (real-slice)



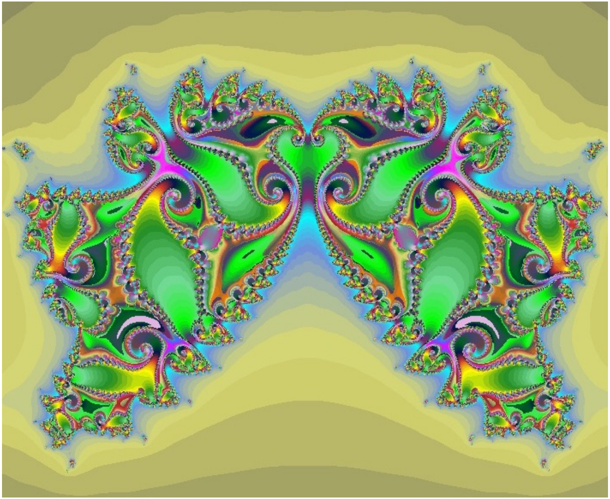
saddle drop picture (real-slice)



etc.

Thank you.



References

[BS1] E. Bedford and J. Smilie. Polynomial diffeomorphisms of $\mathbb{C}^2$: currents, equilibrium measures and hyperbolicity. Invent. Math. **103**(1991), 69-99.
[BS2] E.Bedford and J. Smilie. Polynomial diffeomorphisms of $\mathbb{C}^2$. II : stable manifolds and recurrence. J. Amer. Math. Soc. **4**(1991), no. 4, 657-679.
[BSU] E.Bedford, J. Smilie, T. Ueda. Semi-parabolic Bifurcations in Complex Dimension Two. Commun. Math. Phys. Published online: 30 January 2017.