Hénon dynamics in $\mathbb{C}^2$

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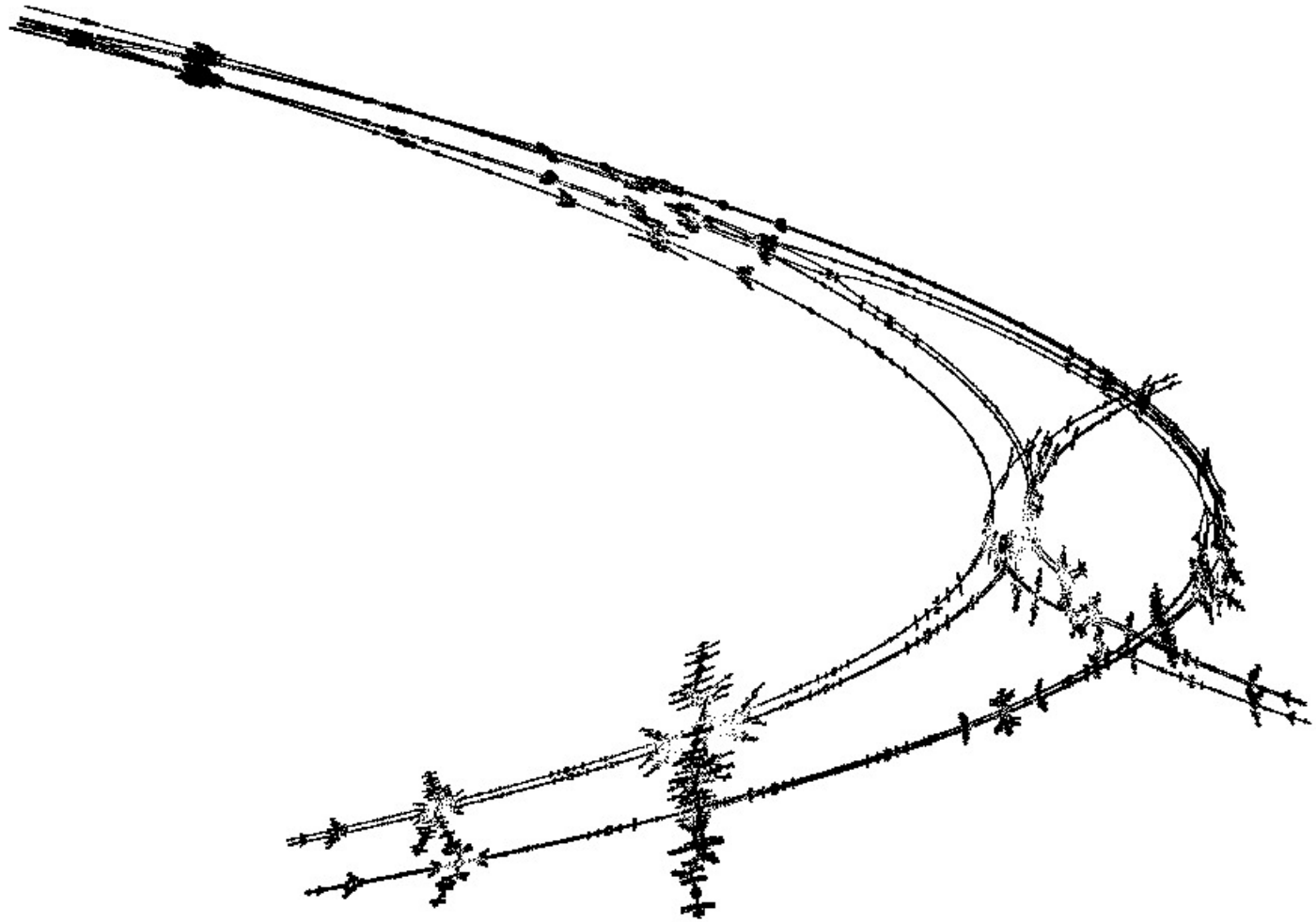
Classical strange attractor of Hénon map

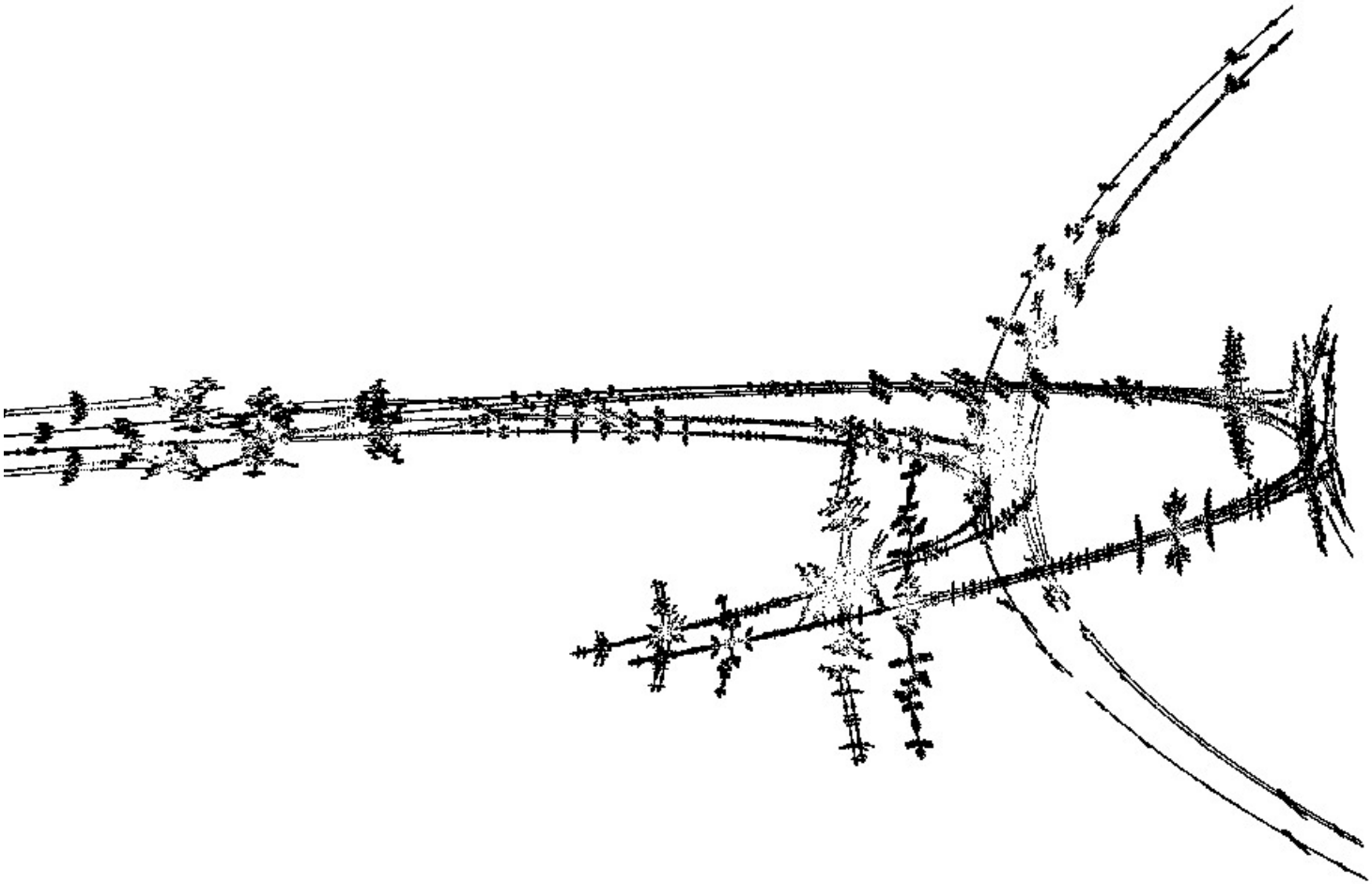
As is well known, Hénon's strange attractor lives in $\mathbb{R}^2$. In this note, the Hénon map $(x, y) \mapsto (X, Y)$ is defined by the following formula.

$$\begin{cases} X &= x^2 + c + by \\ Y &= x \end{cases}$$

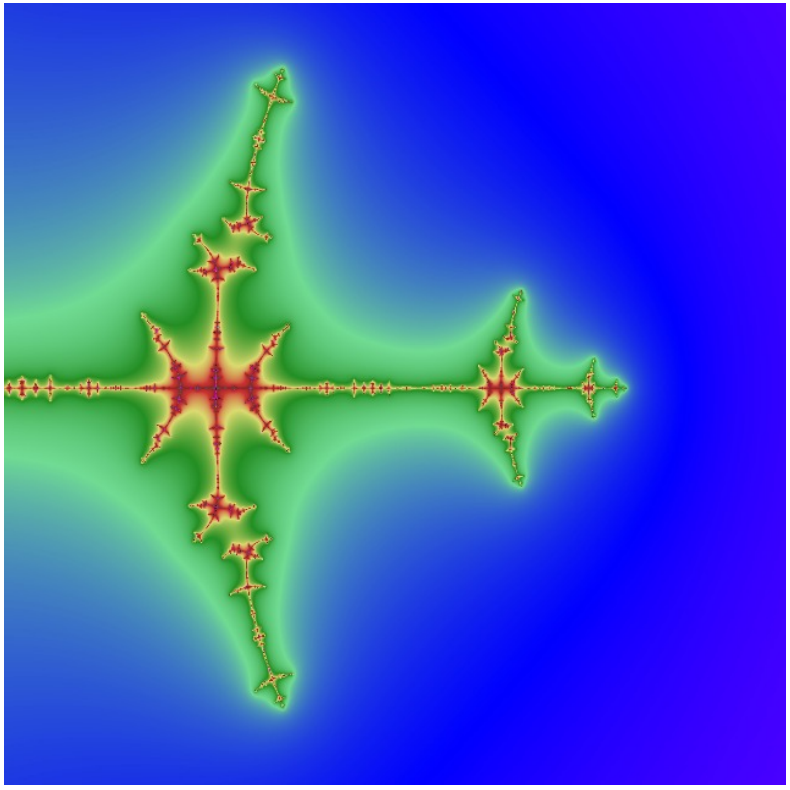
Here, parameter c corresponds to $-a$ in the classical Hénon's family, and the coordinates x, y are rescaled so that we can compare the behavior of the dynamics with the one-dimensional Mandelbrot family of quadratic functions. For most parameters, there are two fixed points., which will be denoted as P and Q.

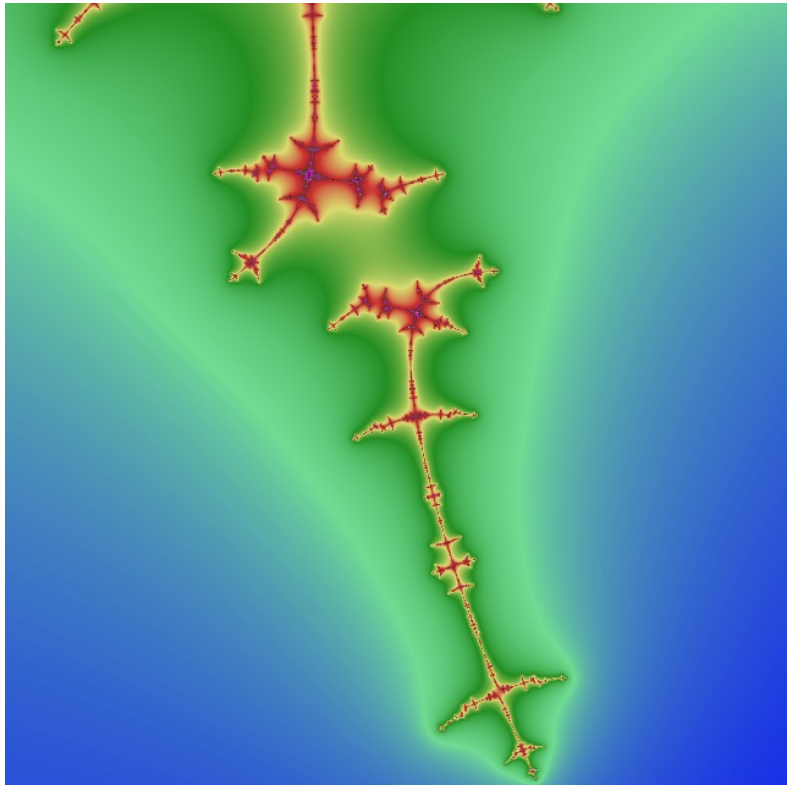


Observe “fish bone” like branches coming out from the turning points of the real strange attractor.

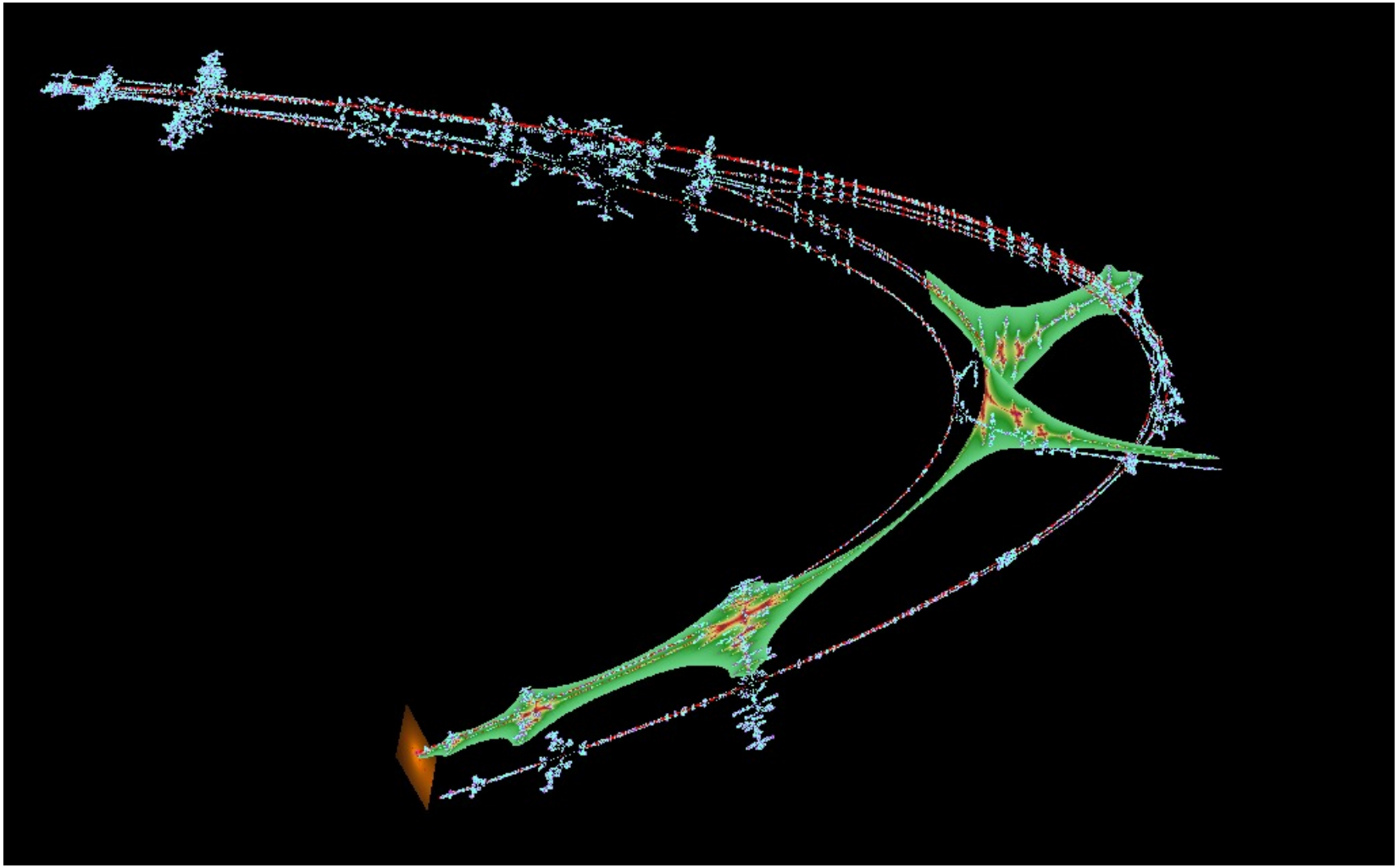


There is a gap between the right upper components and the rest of the set.

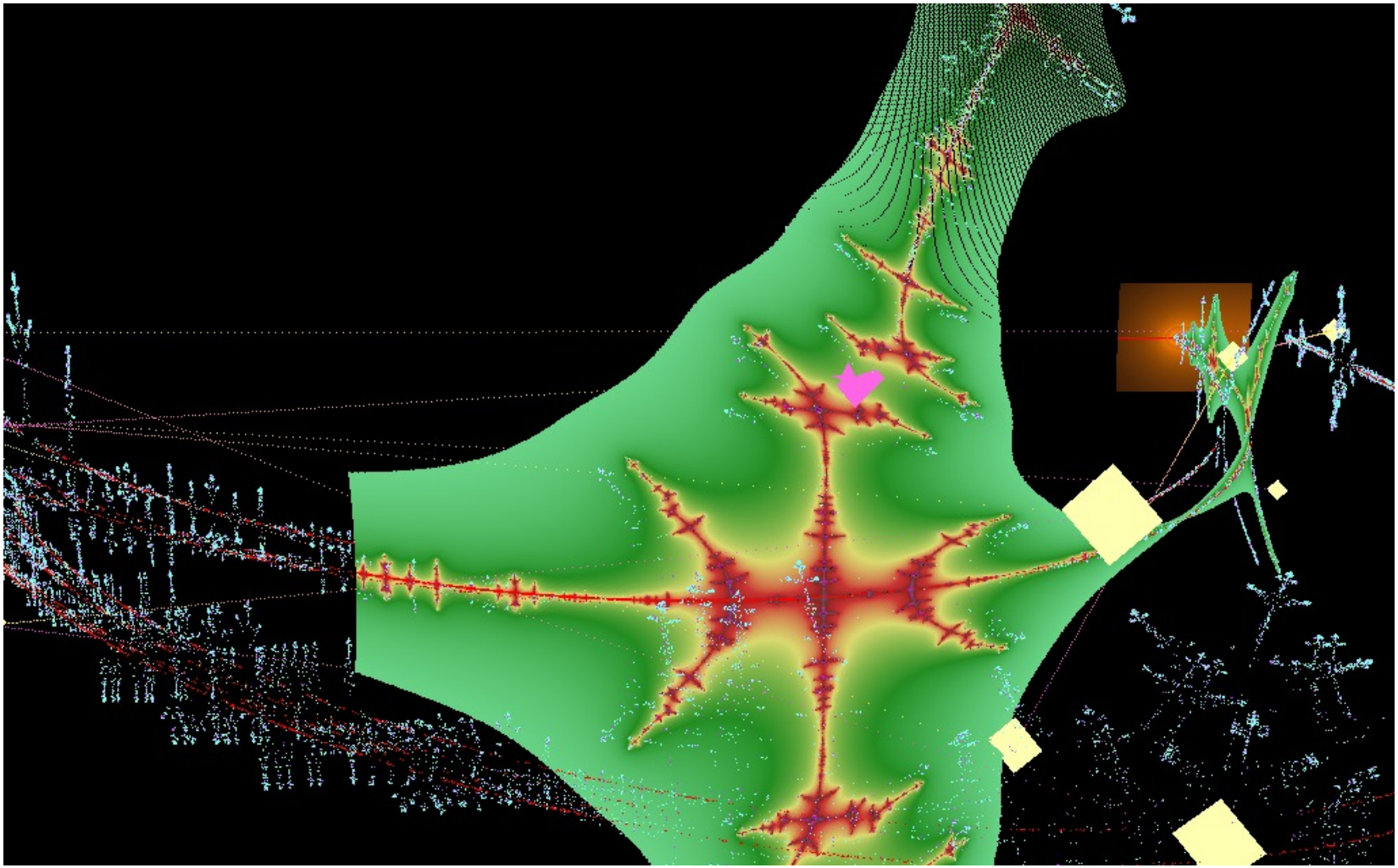




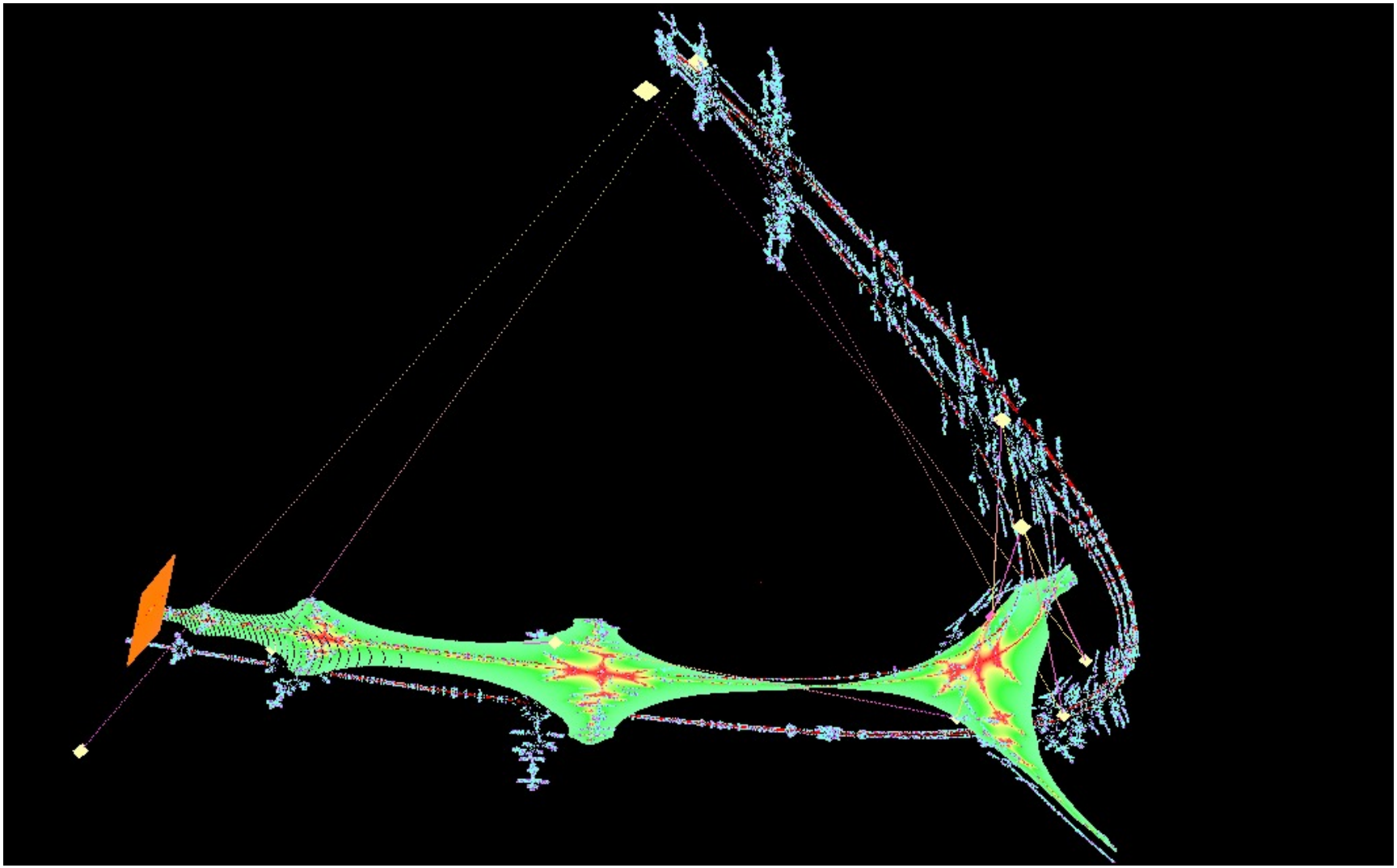
The unstable manifold of P, in the domain of definition of Poincaré's function $\varphi : \mathbb{C} \to \mathbb{C}^2$, colored according to the value of the Green's function.



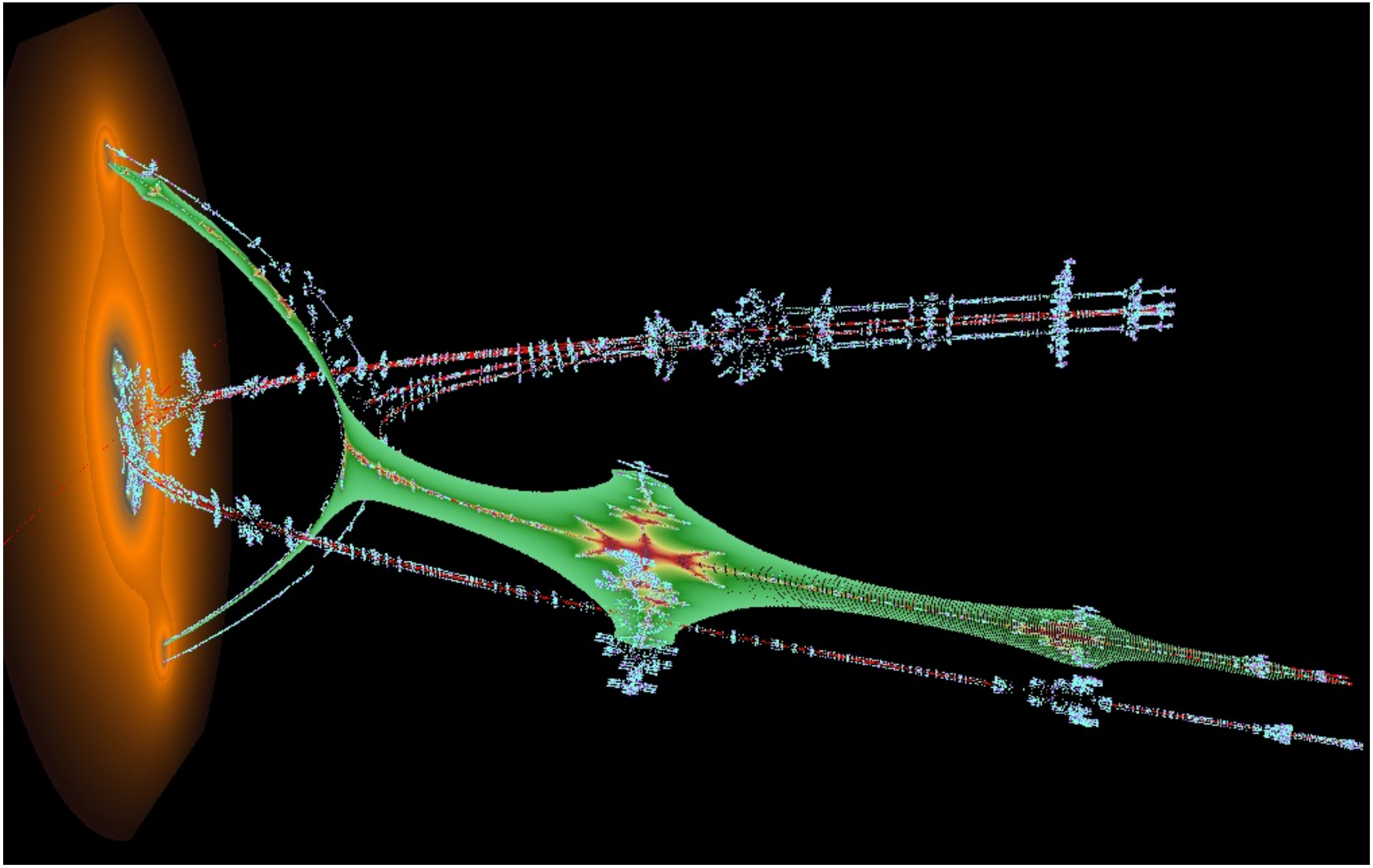
Stable manifold of P, trimmed along a certain level of the value of Green's function is embedded in $\mathbb{C}^2$.



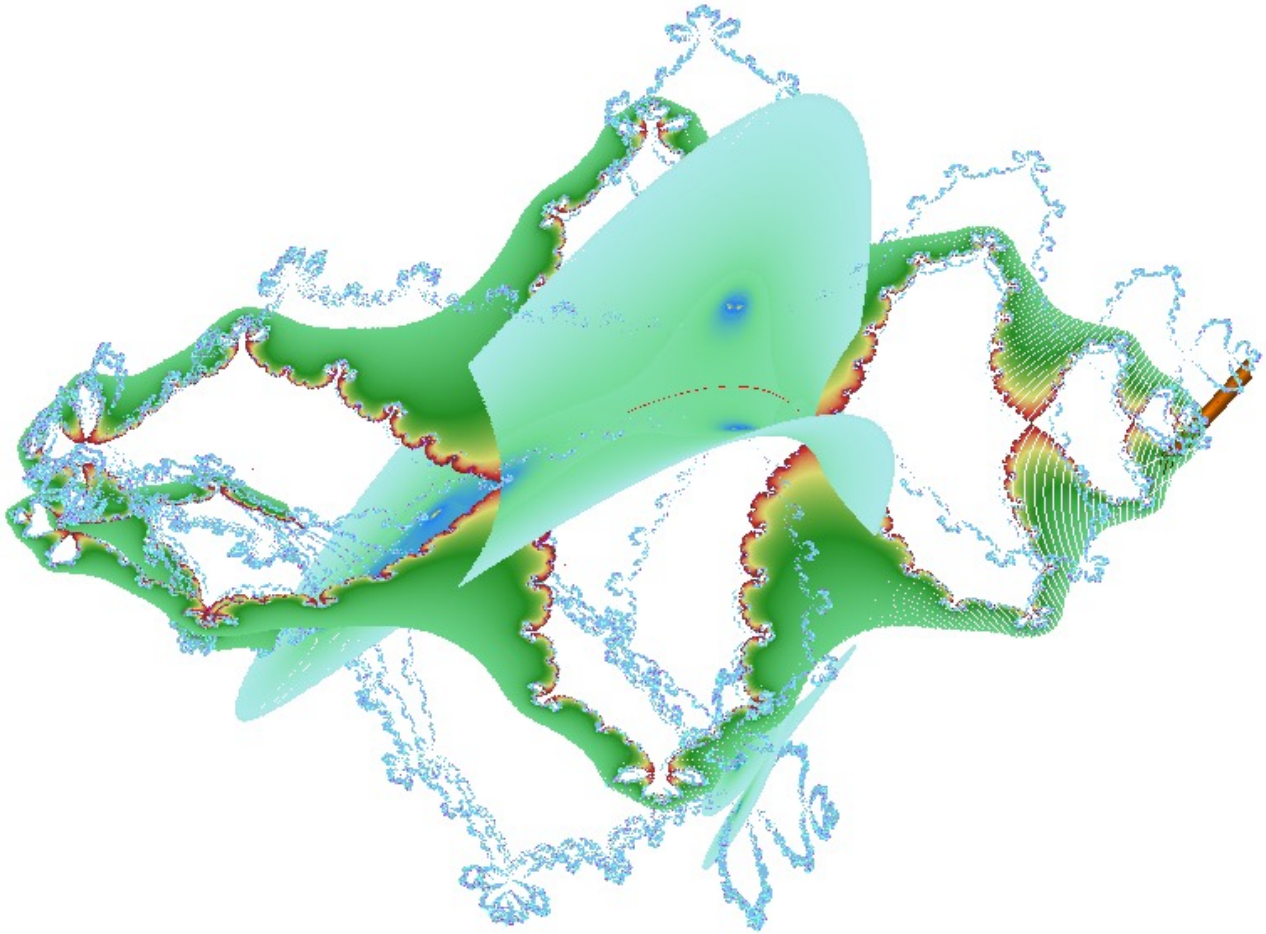
Take a point in the middle of the “canal”.

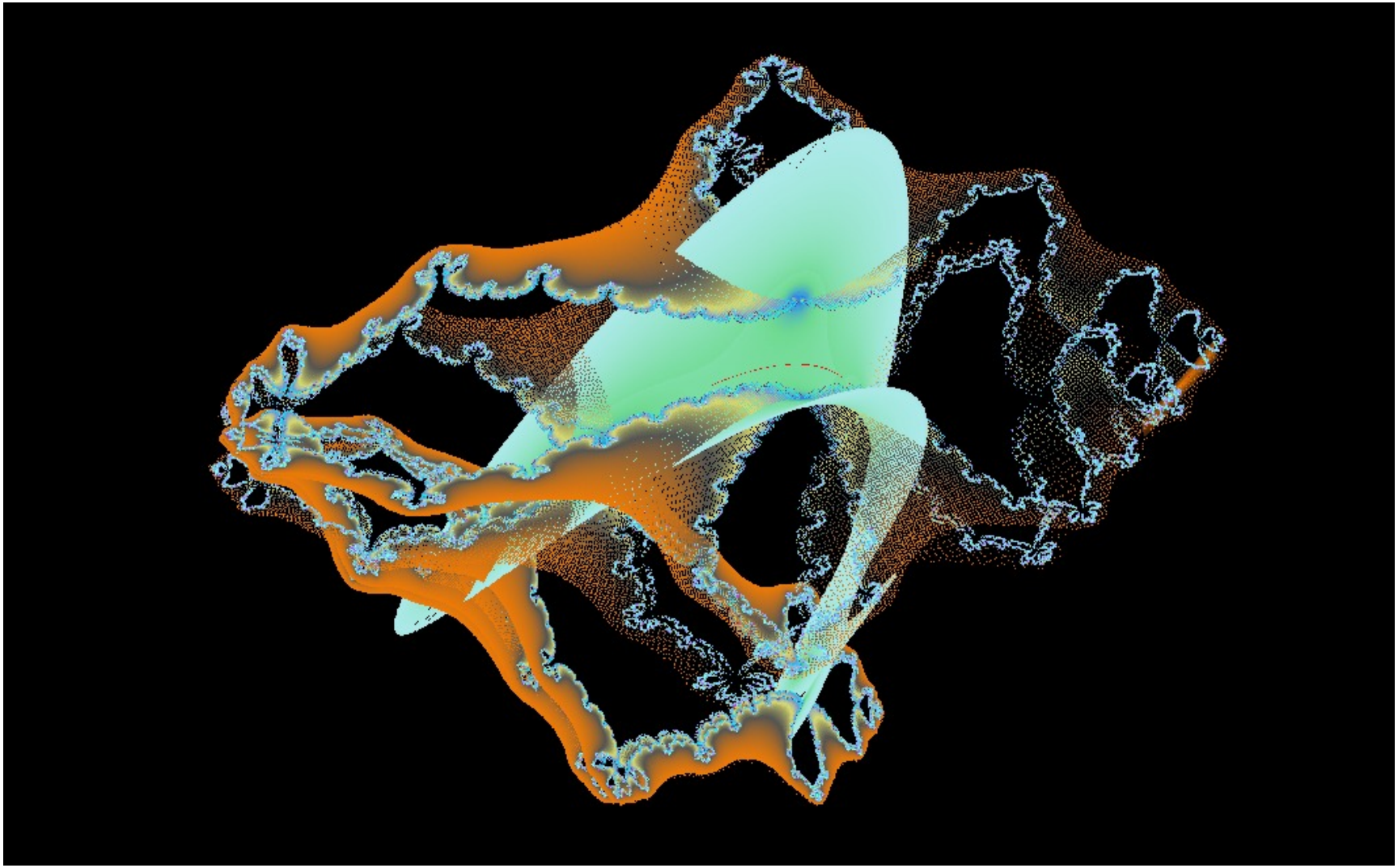


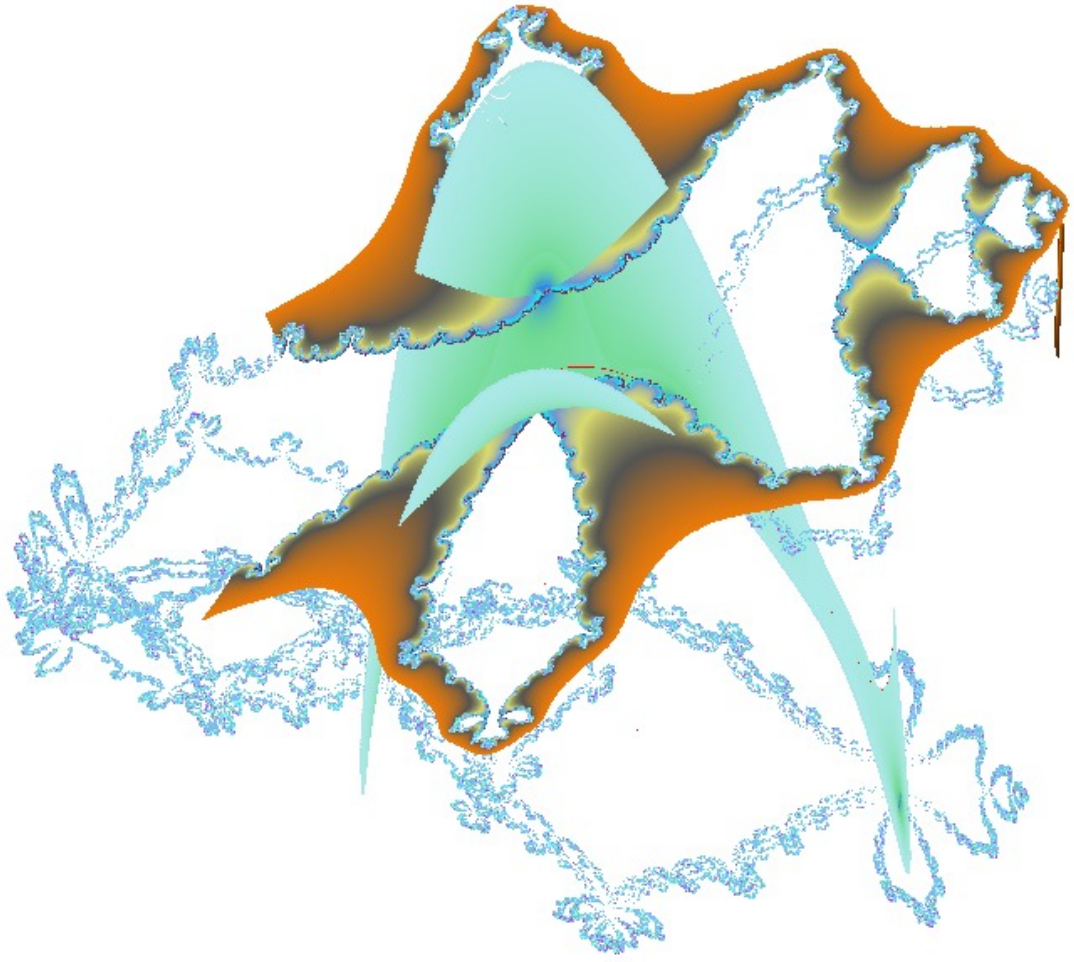
Orbit of the critical point.

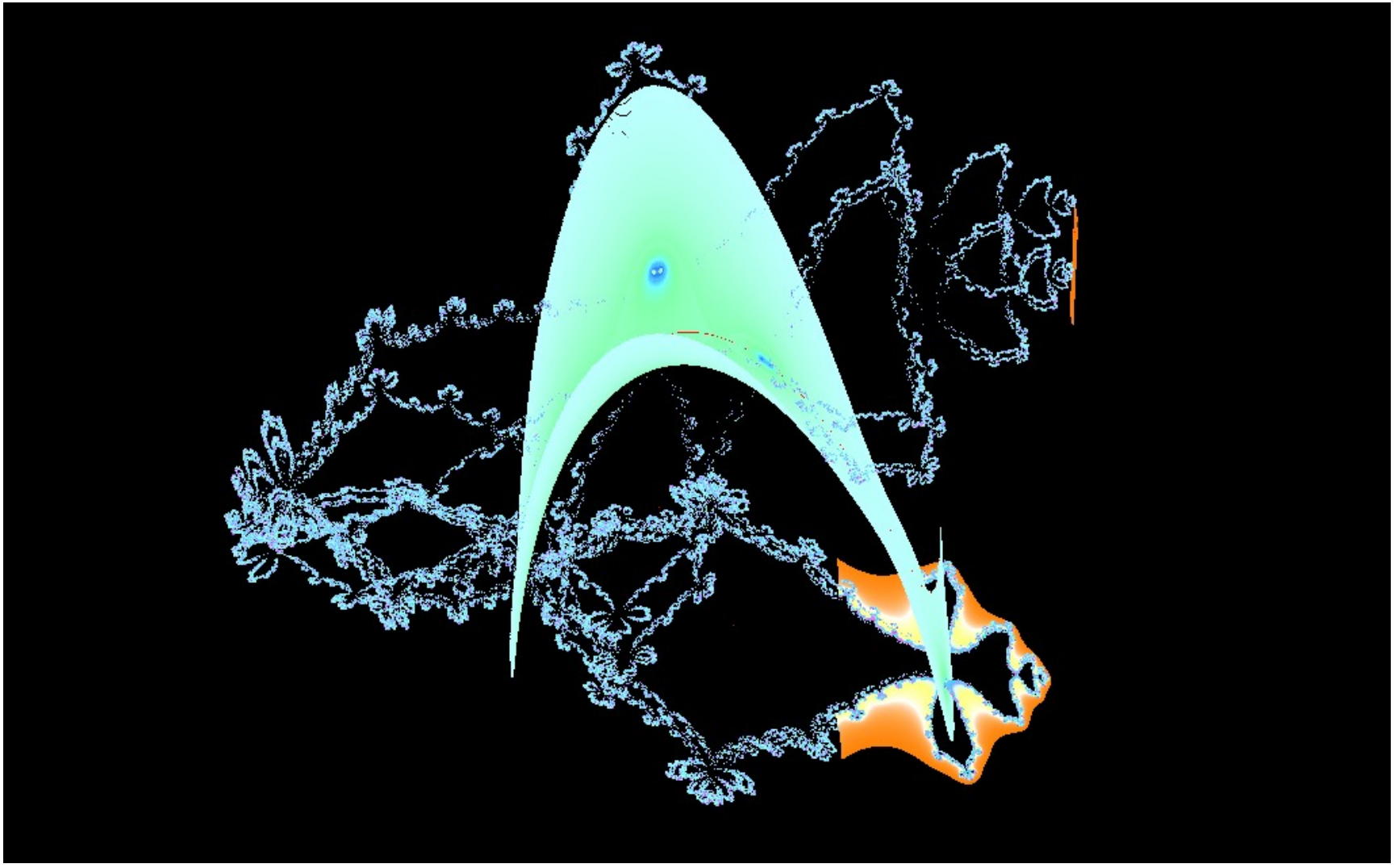


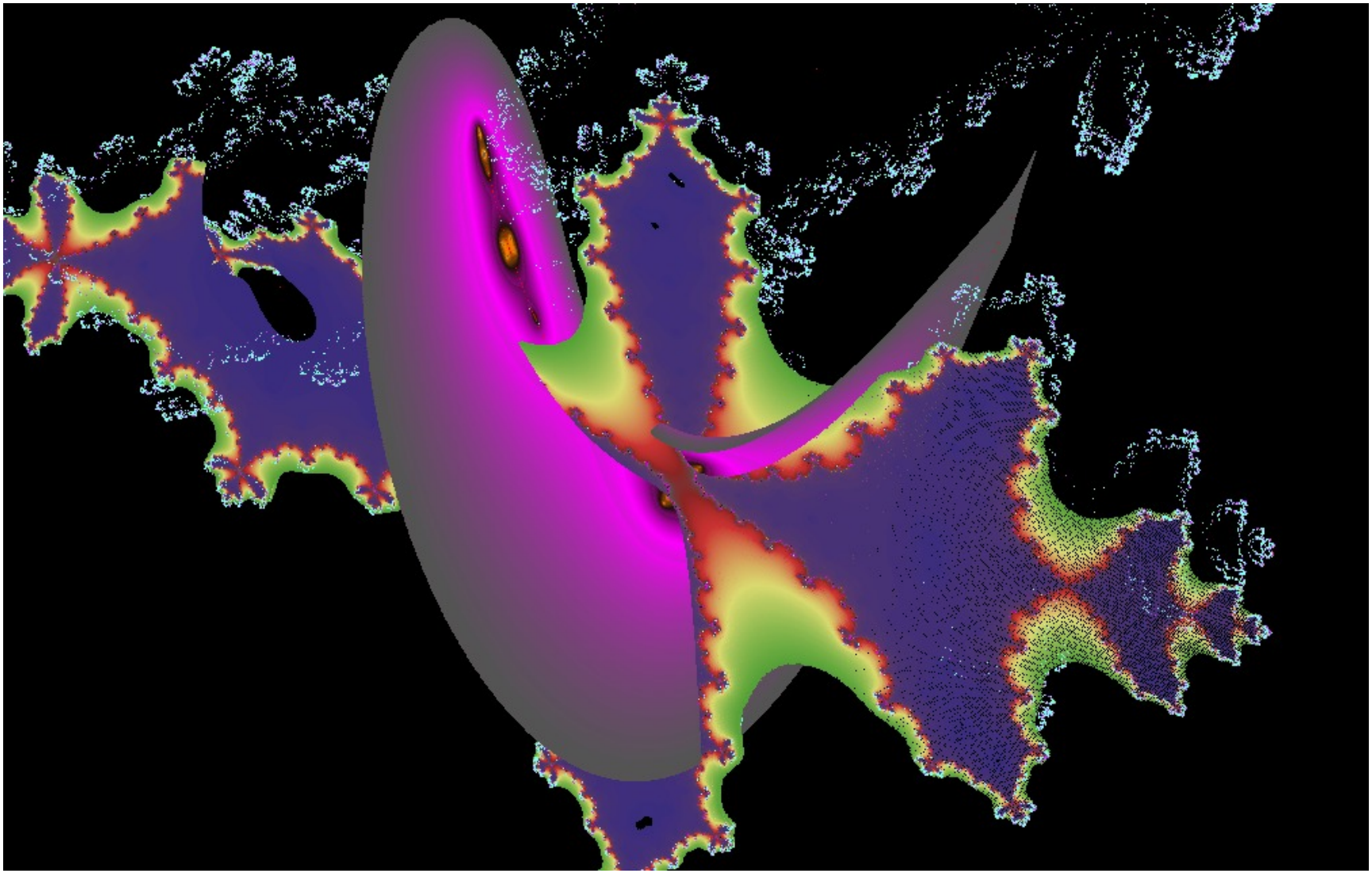
Homoclinic points and heteroclinic points

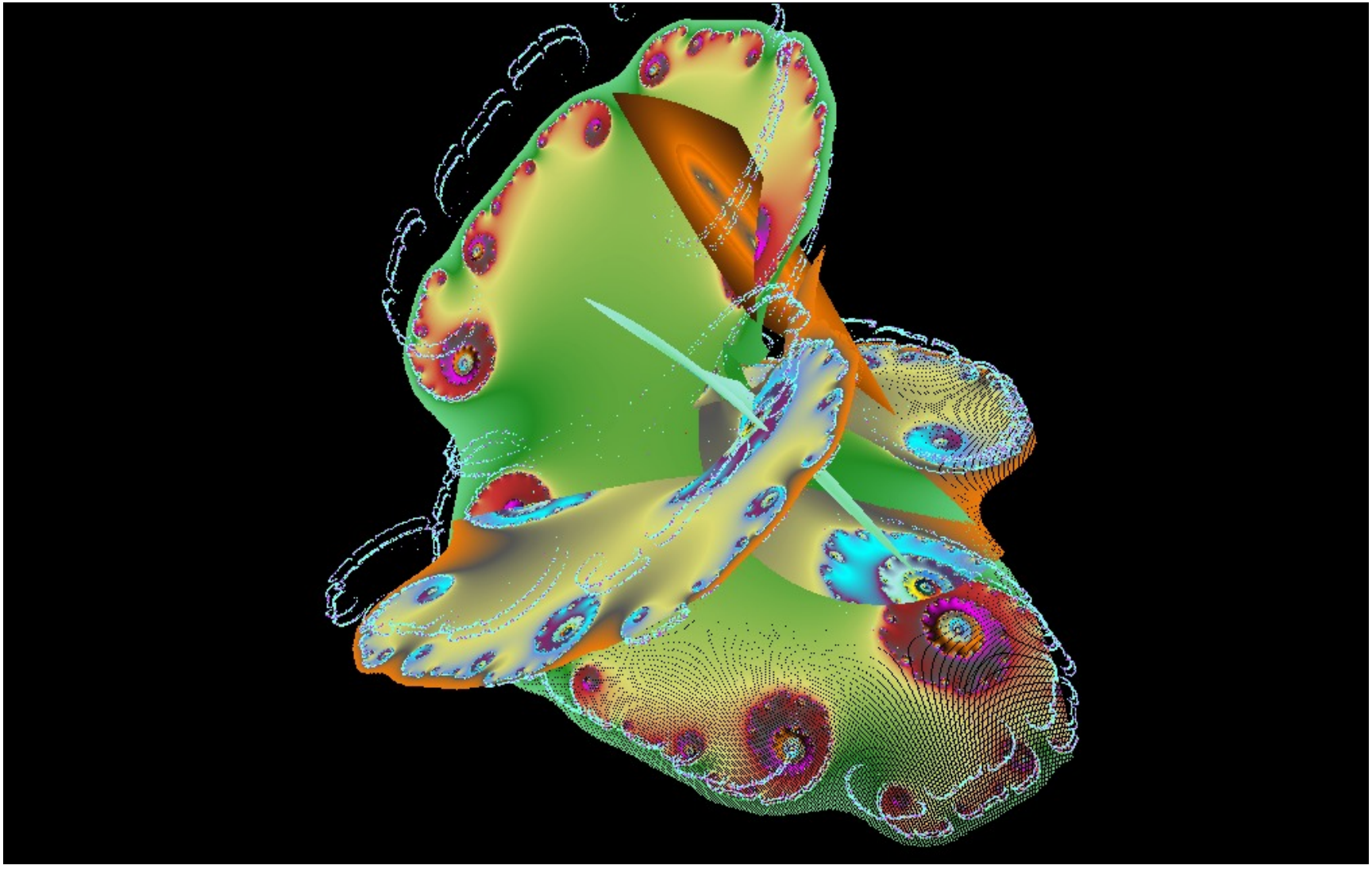












References

[1] E. Bedford, M.Lyubich, and J.Smilie, Distribution of periodic points of polynomial diffeomorphisms of $\mathbb{C}^2$, Invent. Math. **114** (1993), 277-288.
[2] O.Biham, W.Wenzel : Unstable periodic orbits and the symbolic dynamics of the complex Hénon map, Phys. Rev. A42(1990), 4639-4646.
[3] J.H.Hubbard, The Hénon mapping in the complex domain, Chaotic dynamics and fractals (Atlanta, Ga.,1985), Academic Press, Orlando, FL, 1986, 101-111.
[4] H. Poincaré : Sur une classe nouvelle des transcendantes uniformes, Journal de Mathématiques, 4^e série, tôme VI, Fasc. IV(1890).