An elementary approach to invariant cubic curves of surface automorphisms

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Bbirational automorphism $F(u,v) = (v, \frac{\tau v - \delta u}{\tau u + 1})$ has an invariant cubic curve for special values of parameters. These formulas are essentially obtained by E.Bedford and KH. Kim.

Theorem A. The equation of invariant cubic polynomial has solutions (up to a constant multiple) in the following cases.

(Γ_1) $\tau = t^2 + t^3$, $\delta = t^5$, $P_1(u,v) = uv(tu+v) + \frac{1+t+t^2}{1+t}(tu - \frac{v}{t})^2$.

(Γ_2) $\tau = -t - t^2$, $\delta = t^3$, $P_2(u,v) = (tu+v)((1+t)uv + tu + v)$.

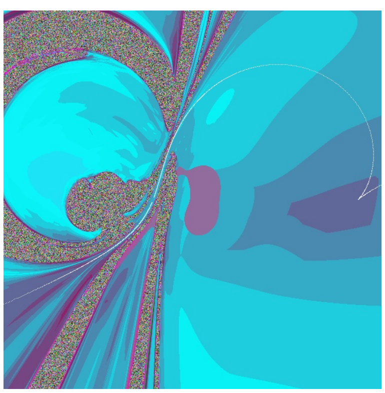
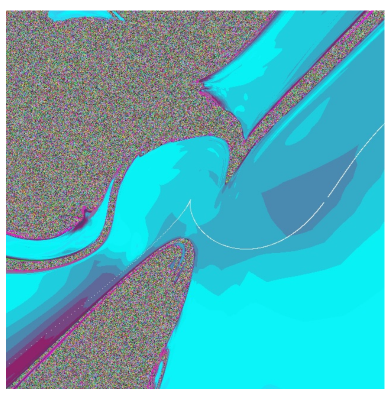
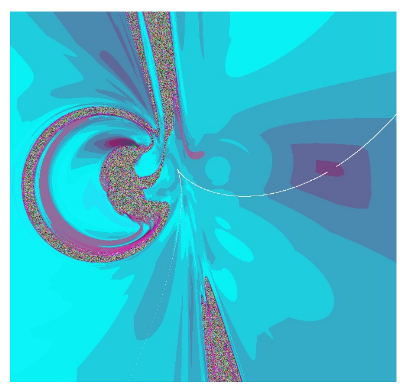
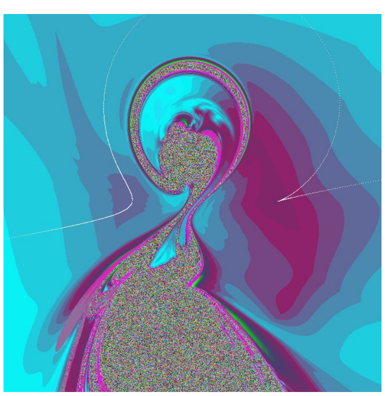
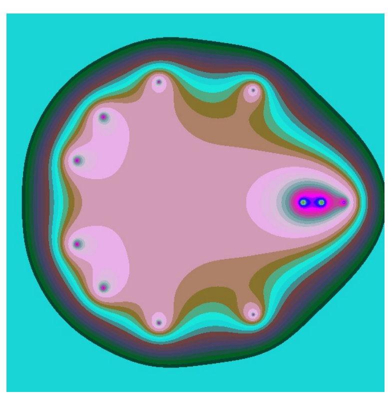
(Γ_3) $\tau = -t$, $\delta = t^2$, $P_3(u,v) = uv(tu+v)$.

Theorem B. Uniformizing functions can be taken as follows.

$$(\Gamma_1) \quad \psi_C(\zeta) = \left(\frac{\xi_0 \zeta^2}{(\zeta+1)(\zeta+t)}, \frac{\xi_0 \zeta^2}{(\zeta+1)(\zeta+\frac{1}{t})} \right), \; \xi_0 = \frac{(1-t)(t^3-1)}{t^2}.$$

$$(\Gamma_2) \quad \psi_L(\zeta) = \left(\frac{\zeta}{\zeta+1} \frac{1-t}{t}, \frac{\zeta}{\zeta+1}(t-1) \right), \quad \psi_Q(\zeta) = \left(\frac{t^{-1}\zeta}{t^{-1}\zeta+1}(t-1), \frac{t\zeta}{t\zeta+1} \frac{1-t}{t} \right).$$

$$(\Gamma_3) \quad \psi_0(\zeta) = \left(\frac{\zeta}{\zeta+1} \frac{t^3-1}{t^2}, \frac{\zeta}{\zeta+1} \frac{1-t^3}{t} \right), \; \psi_1(\zeta) = \left(\frac{t^{-1}\zeta}{t^{-1}\zeta+1} \frac{1-t^3}{t}, 0 \right), \; \psi_2(\zeta) = \left(0, \frac{t\zeta}{t\zeta+1} \frac{t^3-1}{t^2} \right).$$

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