Workshop on the arithmetic geometry of Shimura varieties, representation theory, and related topics

プログラム

７月１８日（水）

9:30–10:30　伊藤哲史（京大数学教室）

Rapoport-Zink spaces and endoscopy

10:45–11:45　今井直毅（京大数理研）

Geometric realization of the local Langlands correspondence for representations of conductor three

13:45–14:45　Keerthi Madapusi Pera (Harvard)

(Talk 1) Integral canonical models for Shimura varieties of Hodge type

15:15–16:15　Gabriel Dospinescu (Ecole Polytechnique)

(Talk 1) Overview of the p-adic Langlands correspondence for $\mathrm{GL}_2(\mathbb{Q}_p)$

16:30–17:30　中村健太郎（北大理）

A generalization of Kato's local ε conjecture for (φ, Γ)-modules over the Robba ring

７月１９日（木）

9:30–10:30　Florian Herzig (Toronto)

(Talk 1) Ordinary representations of $\mathrm{GL}_n(\mathbb{Q}_p)$ and fundamental algebraic representations, I

10:45–11:45　Keerthi Madapusi Pera (Harvard)

(Talk 2) Regular models for Shimura varieties of orthogonal type and the Tate conjecture for $K3$ surfaces in odd characteristic

13:45–14:45　Gabriel Dospinescu (Ecole Polytechnique)

(Talk 2) Locally analytic Jacquet modules and p-adic differential equations

15:15–16:15　Kai-Wen Lan (Minnesota)

(Talk 1) Compactifications of PEL-type Shimura varieties and Kuga families with ordinary loci

16:30–17:30　松本雄也（東大数理）

Good reduction criterion for $K3$ surfaces

７月２０日（金）

9:30–10:30　Florian Herzig (Toronto)

(Talk 2)　Ordinary representations of $\mathrm{GL}_n(\mathbb{Q}_p)$ and fundamental algebraic representations, II

10:45–11:45　Benjamin Schraen (CNRS/UVSQ)

(Talk 1)　Completed cohomology of the tower of modular curves and local-global compatibility

12:00–13:00　Tasho Kaletha (Princeton/IAS)

(Talk 1)　Construction of supercuspidal L-packets for tamely-ramified groups, I

Discussion/Free Afternoon

７月２１日（土）

9:30–10:30　Benjamin Schraen (CNRS/UVSQ)

(Talk 2)　The cohomology of local systems on quotient of Drinfel'd upper half spaces

10:45–11:45　Tasho Kaletha (Princeton/IAS)

(Talk 2)　Construction of supercuspidal L-packets for tamely-ramified groups, II

13:45–14:45　Kai-Wen Lan (Minnesota)

(Talk 2)　Galois representations for regular algebraic automorphic forms over CM fields

15:15–16:15　Przemyslaw Chojecki (Université Pierre et Marie Curie)

Mod p non-abelian Lubin-Tate theory

16:30–17:30　Xu Shen (Orsay)

Cell decomposition and Lefschetz trace formula for some unitary group Rapoport-Zink spaces

７月２２日（日）

9:30–10:30　三枝洋一（京大白眉センター／数学教室）

Potentially good reduction loci of open Shimura varieties and their ℓ-adic cohomology

10:45–11:45　津嶋貴弘（九大数理）

Conjecture on the stable reduction of the Lubin-Tate curve and the Lusztig theory over finite rings

12:00–13:00　Sug Woo Shin (MIT)

Integral Manin problem with PEL structure

Workshop on the arithmetic geometry of Shimura varieties, representation theory, and related topics

講演題目・アブストラクト

講演者　**Przemyslaw Chojecki (Université Pierre et Marie Curie)**

講演題目　**Mod p non-abelian Lubin-Tate theory**

アブストラクト　I will show that the mod p etale cohomology of the Lubin-Tate tower realizes the mod p local Langlands correspondence and a mod p Jacquet-Langlands correspondence. This will be done by comparing the cohomology of the Lubin-Tate tower with the cohomology of the tower of modular curves and using results of Emerton with some further representation-theoretic analysis. I will show two possible notions of a mod p Jacquet-Langlands correspondence and I will discuss a connection of them with a local-global compatibility part of the Buzzard-Diamond-Jarvis conjecture.

講演者　**Gabriel Dospinescu (Ecole Polytechnique)**

講演題目　**(Talk 1) Overview of the p-adic Langlands correspondence for $\mathrm{GL}_2(\mathbb{Q}_p)$**

アブストラクト　We will try to recall in an accessible manner the main objects and results in the p-adic Langlands correspondence, due to Colmez. We will also try to relate them to more recent work of Paskunas.

講演題目　**(Talk 2) Locally analytic Jacquet modules and p-adic differential equations**

アブストラクト　We will show that simple p-adic differential equations control the Jacquet modules of the unitary p-adic principal series of $\mathrm{GL}_2(\mathbb{Q}_p)$. This leads to a very direct proof of conjectures of Berger, Breuil and Emerton. We will try to discuss the relationship between our approach and that of Liu, Xie, Zhang and Colmez.

講演者　**Florian Herzig (Toronto)**

講演題目　**(Talk 1 & 2) Ordinary representations of $\mathrm{GL}_n(\mathbb{Q}_p)$ and fundamental algebraic representations**

アブストラクト　Motivated by a hypothetical p-adic Langlands correspondence for $\mathrm{GL}_n(\mathbb{Q}_p)$ we associate to an n-dimensional ordinary (i.e. upper-triangular) representation ρ of $\mathrm{Gal}(\overline{\mathbb{Q}}_p/\mathbb{Q}_p)$ over E a unitary Banach space representation $\Pi(\rho)^{\mathrm{ord}}$ of $\mathrm{GL}_n(\mathbb{Q}_p)$ over E that is built out of principal series representations. (Here, E is a finite extension of $\mathbb{Q}_p$.) There is an analogous construction over $\overline{\mathbb{F}}_p$. In the latter case we show under suitable hypotheses that $\Pi(\rho)^{\mathrm{ord}}$ occurs in the ρ-part of the cohomology of a compact unitary group. This is joint work with Christophe Breuil. In the first talk we will discuss some relevant background on mod p and p-adic Langlands, Serre weights, and mod p smooth representations of p-adic groups.

講演者　今井直毅（京大数理研）

講演題目　**Geometric realization of the local Langlands correspondence for representations of conductor three**

アブストラクト　We will prove that the local Langlands correspondence for two-dimensional representations of Weil groups of conductor three is realized in the cohomology of one-dimensional Lubin-Tate spaces by studying stable models of Lubin-Tate spaces. In this talk, we will put emphasis on the case where a base field is dyadic. This is a joint work with Takahiro Tsushima.

講演者　伊藤哲史（京大数学教室）

講演題目　**Rapoport-Zink spaces and endoscopy**

アブストラクト　Rapoport-Zink spaces are generalizations of the Lubin-Tate spaces and the Drinfeld upper half spaces, whose ℓ-adic cohomology is expected to realize the local Langlands correspondences and local Jacquet-Langlands correspondences. It is proved by Harris-Taylor, Boyer, Faltings, Fargues,... for the Lubin-Tate spaces and the Drinfeld upper half spaces, but the general case is widely open. Accoring to the work of Boyer for $\mathrm{GL}(n)$, and the speaker and Yoichi Mieda for $\mathrm{GSp}(4)$ and $\mathrm{GU}(3)$, it turned out that a naive expectation should be modified in order to treat local A-packets and L-packets. We try to discuss possible description of the endoscopic decomposition of the ℓ-adic cohomology of the Rapoport-Zink spaces ("Mirror Symmetry for Rapoport-Zink spaces"). This talk also aims to introduce basic notations and definitions on Shimura varieties, Rapoport-Zink spaces, and local/global Langlands correspondences which appear in other lectures in this workshop.

講演者　Tasho Kaletha (Princeton/IAS)

講演題目　**(Talk 1 & 2) Construction of supercuspidal L-packets for tamely-ramified groups**

アブストラクト　We will report on recent work to construct L-packets on general tamely-ramified reductive p-adic groups consisting of supercuspidal representations of low ramification. In the first talk, we will recall some background material on endoscopy and state precise versions of the local Langlands conjecture and endoscopic transfer conjecture, which are adapted to groups that are not necessarily quasi-split. In the second talk, we will state what is currently known for general groups and discuss some of the details of the construction.

講演者　Kai-Wen Lan (Minnesota)

講演題目　**(Talk 1) Compactifications of PEL-type Shimura varieties and Kuga families with ordinary loci**

アブストラクト　I will explain the constructions of normal flat p-integral models of various algebraic compactifications of PEL-type Shimura varieties and Kuga families, allowing ramification (including levels) at p, with good behaviors over the loci where certain (multiplicative) ordinary level structures are defined. (Explicitly, in the case of $\mathrm{GSp}(2n)$/totally real or $\mathrm{GU}(n, n)$/CM, for

example, we allow arbitrary ramifications of p in the totally real or CM field, and allow level structures as deep as $\Gamma_1(p^m)$ for arbitrary m.) These constructions (in the case of GU(n,n)/CM) are used in my joint work with Michael Harris, Richard Taylor, and Jack Thorne on attaching overconvergent cusp forms and Galois representations to regular algebraic automorphic representations of GL(n) over CM fields, without any hypotheses on self-duality.

講演題目 (Talk 2) Galois representations for regular algebraic automorphic forms over CM fields

アブストラクト I will report on my joint work with Michael Harris, Richard Taylor, and Jack Thorne on the construction of p-adic Galois reprensentations for regular algebraic cuspidal automorphic representations of GL(n) over CM (or totally real) fields, without any self-duality hypothesis.

I will focus on a crucial new idea, namely the construction of overconvergent cusp forms with desired Hecke actions using the partial toroidal boundary of the ordinary loci of unitary Shimura varieties and Kuga families over them. I will outline other needed steps for the construction of the desired Galois representations (for automorphic representations of GL(n)) using such overconvergent cusp forms, and give more details if time permits.

I would like to emphasize that it is believed that such Galois representations do not appear in the p-adic etale cohomology of Shimura varieties and Kuga families over them, at least not in a way detectable by counting points in characteristic p. Instead, we construct such Galois representations using p-adic limits of known ones, and this is why the construction of overconverngent cusp forms with desired Hecke actions is crucial, as they can be viewed as p-adic limits of classical cusp forms (for which people do know how to attach Galois representations, thanks to previous works with self-duality hypotheses).

講演者 Keerthi Madapusi Pera (Harvard)

講演題目 (Talk 1) Integral canonical models for Shimura varieties of Hodge type

アブストラクト I will give an overview of the construction of integral models for Shimura varieties of Hodge type via the methods of Kisin and Faltings (as well as Vasiu). Since these Shimura varieties have no moduli interpretation, the methods are necessarily ad hoc, involving Deligne's theory of absolute Hodge cycles, p-adic Hodge theory and the deformation theory for p-divisible groups. I will quickly review these, and indicate the key difficulties in the construction.

講演題目 (Talk 2) Regular models for Shimura varieties of orthogonal type and the Tate conjecture for $K3$ surfaces in odd characteristic

アブストラクト I will show how to construct good integral models for orthogonal Shimura varieties even at (odd) primes where the discriminant is not invertible. Then we will see that the certain moduli spaces of polarized $K3$ surfaces can be viewed as open sub-schemes of these integral models. This, combined with recent results of Kisin towards the Langlands-Rapoport conjecture, will lead us to a proof of the Tate conjecture in odd characteristic p for $K3$ surfaces admitting a polarization of degree indivisible by p^2. The same methods also prove the Tate conjecture for cubic fourfolds in odd characteristic.

講演者　松本雄也（東大数理）

講演題目　Good reduction criterion for $K3$ surfaces

アブストラクト　Abelian varieties over local fields have good reduction if and only if their l-adic etale cohomology (equivalently Tate modules) are unramified [Serre–Tate]. I prove a similar result for (potential) good reduction of projective $K3$ surfaces of small degree (relative to the residue characteristic). This result covers wider classes of $K3$ surfaces than those in my previous talks (in Kyoto, Hiroshima, Kyushu).

講演者　三枝洋一（京大白眉センター／数学教室）

講演題目　Potentially good reduction loci of open Shimura varieties and their ℓ-adic cohomology

アブストラクト　If a Shimura variety is compact and has a proper integral model, its ℓ-adic cohomology can be computed by the nearby cycle complex. However, if it is not compact, we do not have such a result in general. In this talk, for a certain class of Shimura varieties, I will introduce rigid open subsets called the potentially good reduction loci. If a Shimura variety has a reasonable integral model, then the ℓ-adic cohomology of this open subset will coincide with the cohomology of the nearby cycle complex. We compare the compactly supported ℓ-adic cohomology of a Shimura variety and that of its potentially good reduction locus. This is a joint work with Naoki Imai.

講演者　中村健太郎（北大理）

講演題目　A generalization of Kato's local ε conjecture for (φ, Γ)-modules over the Robba ring

アブストラクト　In his preprint "Lectures on the approach to Iwasawa theory of Hasse-Weil L-functions via B_{dR}, Part II", Kazuya Kato proposed a conjecture which is called local ε conjecture. This conjecture roughly says that the determinant of the Galois cohomology of a family of p-adic Galois representations has a canonical base which can be described by using Deligne-Langlands ε constants. In our talk, we generalize his conjecture for families of (φ, Γ)-modules over the Robba ring and prove a part of this conjecture in some special cases (more precisely, trianguline case). The two key ingredients are the recent result of Kedlaya-Pottharst-Xiao on the finiteness of cohomologies of (φ, Γ)-modules and my result on Bloch-Kato exponential maps for (φ, Γ)-modules. This talk is a work in progress.

講演者　Benjamin Schraen (CNRS/UVSQ)

講演題目　(Talk 1) Completed cohomology of the tower of modular curves and local-global compatibility

アブストラクト　We will introduce completed p-adic etale cohomology of the tower of modular curves. We will next explain the strategy of Emerton to realize the local p-adic Langlands correspondence for GL2(Qp) in this cohomology.

講演題目　(Talk 2) The cohomology of local systems on quotient of Drinfel'd upper half spaces

アブストラクト　(joint work with Jan Kohlhaase) If X is a d-dimensional variety uniformized by Drinfel'd upper half space, Schneider has conjectured the transversality of two natural filtrations on the de Rham cohomology of local systems on X. We will explain how vanishing theorems for the cohomology of discrete groups on locally analytic representations of GL_{d+1} could be used to prove this conjecture, and effectively prove any new cases.

講演者　**Xu Shen (Orsay)**
講演題目　**Cell decomposition and Lefschetz trace formula for some unitary group Rapoport-Zink spaces**
アブストラクト　We explain that, how to use Fargues's theory of Harder-Narasimhan filtration for finite flat group schemes to find certain compact analytic domain in the basic Rapoport-Zink spaces of signature $(1, n-1)$ for some unitary groups, so that its translations under the two related p-adic reductive groups form a locally finite covering of the whole space. By studying the action of regular elliptic elements and applying Mieda's theorem, we can deduce a Lefschetz trace formula for all large enough analytic domains coming from this covering. This will be sufficient for applications to representation theory.

講演者　**Sug Woo Shin (MIT)**
講演題目　**Integral Manin problem with PEL structure**
アブストラクト　The rational/integral Manin problem asks whether a given p-divisible group over the algebraic closure of $\mathbb{F}_p$ can be realized as the p-divisible group of an abelian variety over the same field up to an isogeny/isomorphism. The analogous problem for p-divisible groups and abelian varieties with PEL structure have been solved by Vasiu and Viehmann-Wedhorn for p-unramified PEL data of type A and C. I will explain a different approach based on Honda-Tate theory and Galois cohomology to obtain the answer without any unramified hypothesis. As an application one obtains a relatively simple argument to show the non-emptiness of Newton strata of Shimura varieties attached to p-unramified PEL data of type A and C.

講演者　津嶋貴弘（九大数理）
講演題目　**Conjecture on the stable reduction of the Lubin-Tate curve and the Lusztig theory over finite rings**
アブストラクト　A connection between the non-abelian Lubin-Tate theory and the Deligne-Lusztig theory seems to be mysterious. T. Yoshida proves that the Deligne-Lusztig variety for GL_h appears in the stable reduction of the Lubin-Tate space of level 1.
In this talk, we construct a curve over a finite field with some group action using Lusztig theory over finite rings and study its cohomology. We state a conjecture that this curve appears in the stable reduction of the Lubin-Tate curve. As an evidence, we prove this conjecture in the case where the level is two. This is a joint work with Tetsushi Ito and Yoichi Mieda.