

Workshop on the arithmetic geometry of Shimura varieties, representation theory, and related topics

Date July 18 (Wed) - 22 (Sun), 2012
Location Department of Mathematics, Hokkaido University
Faculty of Science Building #4, Lecture Room 4-501
Organizers Noriyuki Abe (Hokkaido), Yoichi Mieda (Kyoto), Tetsushi Ito (Kyoto)
Webpage <http://www.math.kyoto-u.ac.jp/~tetsushi/workshop201207/index.html>

Program

July 18 (Wed)

- 9:30–10:30 Tetsushi Ito (Kyoto/Math Dept)
Rapoport-Zink spaces and endoscopy
- 10:45–11:45 Naoki Imai (Kyoto/RIMS)
Geometric realization of the local Langlands correspondence for representations of conductor three
- 13:45–14:45 Keerthi Madapusi Pera (Harvard)
(Talk 1) Integral canonical models for Shimura varieties of Hodge type
- 15:15–16:15 Gabriel Dospinescu (Ecole Polytechnique)
(Talk 1) Overview of the p -adic Langlands correspondence for $GL_2(\mathbb{Q}_p)$
- 16:30–17:30 Kentaro Nakamura (Hokkaido)
A generalization of Kato's local ε conjecture for (φ, Γ) -modules over the Robba ring

July 19 (Thu)

- 9:30–10:30 Florian Herzig (Toronto)
(Talk 1) Ordinary representations of $GL_n(\mathbb{Q}_p)$ and fundamental algebraic representations, I
- 10:45–11:45 Keerthi Madapusi Pera (Harvard)
(Talk 2) Regular models for Shimura varieties of orthogonal type and the Tate conjecture for $K3$ surfaces in odd characteristic
- 13:45–14:45 Gabriel Dospinescu (Ecole Polytechnique)
(Talk 2) Locally analytic Jacquet modules and p -adic differential equations
- 15:15–16:15 Kai-Wen Lan (Minnesota)
(Talk 1) Compactifications of PEL-type Shimura varieties and Kuga families with ordinary loci
- 16:30–17:30 Yuya Matsumoto (Tokyo)
Good reduction criterion for $K3$ surfaces

July 20 (Fri)

- 9:30–10:30 Florian Herzig (Toronto)
(Talk 2) Ordinary representations of $GL_n(\mathbb{Q}_p)$ and fundamental algebraic representations, II
- 10:45–11:45 Benjamin Schraen (CNRS/UVSQ)
(Talk 1) Completed cohomology of the tower of modular curves and local-global compatibility
- 12:00–13:00 Tasho Kaletha (Princeton/IAS)
(Talk 1) Construction of supercuspidal L -packets for tamely-ramified groups, I
Discussion/Free Afternoon

July 21 (Sat)

- 9:30–10:30 Benjamin Schraen (CNRS/UVSQ)
(Talk 2) The cohomology of local systems on quotient of Drinfel'd upper half spaces
- 10:45–11:45 Tasho Kaletha (Princeton/IAS)
(Talk 2) Construction of supercuspidal L -packets for tamely-ramified groups, II
- 13:45–14:45 Kai-Wen Lan (Minnesota)
(Talk 2) Galois representations for regular algebraic automorphic forms over CM fields
- 15:15–16:15 Przemyslaw Chojecki (Université Pierre et Marie Curie)
Mod p non-abelian Lubin-Tate theory
- 16:30–17:30 Xu Shen (Orsay)
Cell decomposition and Lefschetz trace formula for some unitary group Rapoport-Zink spaces

July 22 (Sun)

- 9:30–10:30 Yoichi Mieda (Kyoto/Math Dept & Hakubi center)
Potentially good reduction loci of open Shimura varieties and their ℓ -adic cohomology
- 10:45–11:45 Takahiro Tsushima (Kyushu)
Conjecture on the stable reduction of the Lubin-Tate curve and the Lusztig theory over finite rings
- 12:00–13:00 Sug Woo Shin (MIT)
Integral Manin problem with PEL structure

Workshop on the arithmetic geometry of Shimura varieties, representation theory, and related topics

Abstract

Speaker: Przemyslaw Chojecki (Université Pierre et Marie Curie)

Title: Mod p non-abelian Lubin-Tate theory

Abstract: I will show that the mod p étale cohomology of the Lubin-Tate tower realizes the mod p local Langlands correspondence and a mod p Jacquet-Langlands correspondence. This will be done by comparing the cohomology of the Lubin-Tate tower with the cohomology of the tower of modular curves and using results of Emerton with some further representation-theoretic analysis. I will show two possible notions of a mod p Jacquet-Langlands correspondence and I will discuss a connection of them with a local-global compatibility part of the Buzzard-Diamond-Jarvis conjecture.

Speaker: Gabriel Dospinescu (Ecole Polytechnique)

Title: (Talk 1) Overview of the p -adic Langlands correspondence for $\mathrm{GL}_2(\mathbb{Q}_p)$

Abstract: We will try to recall in an accessible manner the main objects and results in the p -adic Langlands correspondence, due to Colmez. We will also try to relate them to more recent work of Paskunas.

Title: (Talk 2) Locally analytic Jacquet modules and p -adic differential equations

Abstract: We will show that simple p -adic differential equations control the Jacquet modules of the unitary p -adic principal series of $\mathrm{GL}_2(\mathbb{Q}_p)$. This leads to a very direct proof of conjectures of Berger, Breuil and Emerton. We will try to discuss the relationship between our approach and that of Liu, Xie, Zhang and Colmez.

Speaker: Florian Herzig (Toronto)

Title: (Talk 1 & 2) Ordinary representations of $\mathrm{GL}_n(\mathbb{Q}_p)$ and fundamental algebraic representations

Abstract: Motivated by a hypothetical p -adic Langlands correspondence for $\mathrm{GL}_n(\mathbb{Q}_p)$ we associate to an n -dimensional ordinary (i.e. upper-triangular) representation ρ of $\mathrm{Gal}(\overline{\mathbb{Q}}_p/\mathbb{Q}_p)$ over E a unitary Banach space representation $\Pi(\rho)^{\mathrm{ord}}$ of $\mathrm{GL}_n(\mathbb{Q}_p)$ over E that is built out of principal series representations. (Here, E is a finite extension of \mathbb{Q}_p .) There is an analogous construction over $\overline{\mathbb{F}}_p$. In the latter case we show under suitable hypotheses that $\Pi(\rho)^{\mathrm{ord}}$ occurs in the ρ -part of the cohomology of a compact unitary group. This is joint work with Christophe Breuil. In the first talk we will discuss some relevant background on mod p and p -adic Langlands, Serre weights, and mod p smooth representations of p -adic groups.

Speaker: Naoki Imai (Kyoto/RIMS)

Title: Geometric realization of the local Langlands correspondence for representations of conductor three

Abstract: We will prove that the local Langlands correspondence for two-dimensional representations of Weil groups of conductor three is realized in the cohomology of one-dimensional Lubin-Tate spaces by studying stable models of Lubin-Tate spaces. In this talk, we will put emphasis on the case where a base field is dyadic. This is a joint work with Takahiro Tsushima.

Speaker: Tetsushi Ito (Kyoto/Math Dept)

Title: Rapoport-Zink spaces and endoscopy

Abstract: Rapoport-Zink spaces are generalizations of the Lubin-Tate spaces and the Drinfeld upper half spaces, whose ℓ -adic cohomology is expected to realize the local Langlands correspondences and local Jacquet-Langlands correspondences. It is proved by Harris-Taylor, Boyer, Faltings, Fargues,... for the Lubin-Tate spaces and the Drinfeld upper half spaces, but the general case is widely open. According to the work of Boyer for $GL(n)$, and the speaker and Yoichi Mieda for $GSp(4)$ and $GU(3)$, it turned out that a naive expectation should be modified in order to treat local A -packets and L -packets. We try to discuss possible description of the endoscopic decomposition of the ℓ -adic cohomology of the Rapoport-Zink spaces (“Mirror Symmetry for Rapoport-Zink spaces”). This talk also aims to introduce basic notations and definitions on Shimura varieties, Rapoport-Zink spaces, and local/global Langlands correspondences which appear in other lectures in this workshop.

Speaker: Tasho Kaletha (Princeton/IAS)

Title: (Talk 1 & 2) Construction of supercuspidal L -packets for tamely-ramified groups

Abstract: We will report on recent work to construct L -packets on general tamely-ramified reductive p -adic groups consisting of supercuspidal representations of low ramification. In the first talk, we will recall some background material on endoscopy and state precise versions of the local Langlands conjecture and endoscopic transfer conjecture, which are adapted to groups that are not necessarily quasi-split. In the second talk, we will state what is currently known for general groups and discuss some of the details of the construction.

Speaker: Kai-Wen Lan (Minnesota)

Title: (Talk 1) Compactifications of PEL-type Shimura varieties and Kuga families with ordinary loci

Abstract: I will explain the constructions of normal flat p -integral models of various algebraic compactifications of PEL-type Shimura varieties and Kuga families, allowing ramification (including levels) at p , with good behaviors over the loci where certain (multiplicative) ordinary level

structures are defined. (Explicitly, in the case of $\mathrm{GSp}(2n)/\text{totally real}$ or $\mathrm{GU}(n, n)/\mathrm{CM}$, for example, we allow arbitrary ramifications of p in the totally real or CM field, and allow level structures as deep as $\Gamma_1(p^m)$ for arbitrary m .) These constructions (in the case of $\mathrm{GU}(n, n)/\mathrm{CM}$) are used in my joint work with Michael Harris, Richard Taylor, and Jack Thorne on attaching overconvergent cusp forms and Galois representations to regular algebraic automorphic representations of $\mathrm{GL}(n)$ over CM fields, without any hypotheses on self-duality.

Title: (Talk 2) Galois representations for regular algebraic automorphic forms over CM fields

Abstract: I will report on my joint work with Michael Harris, Richard Taylor, and Jack Thorne on the construction of p -adic Galois representations for regular algebraic cuspidal automorphic representations of $\mathrm{GL}(n)$ over CM (or totally real) fields, without any self-duality hypothesis.

I will focus on a crucial new idea, namely the construction of overconvergent cusp forms with desired Hecke actions using the partial toroidal boundary of the ordinary loci of unitary Shimura varieties and Kuga families over them. I will outline other needed steps for the construction of the desired Galois representations (for automorphic representations of $\mathrm{GL}(n)$) using such overconvergent cusp forms, and give more details if time permits.

I would like to emphasize that it is believed that such Galois representations do not appear in the p -adic étale cohomology of Shimura varieties and Kuga families over them, at least not in a way detectable by counting points in characteristic p . Instead, we construct such Galois representations using p -adic limits of known ones, and this is why the construction of overconvergent cusp forms with desired Hecke actions is crucial, as they can be viewed as p -adic limits of classical cusp forms (for which people do know how to attach Galois representations, thanks to previous works with self-duality hypotheses).

Speaker: Keerthi Madapusi Pera (Harvard)

Title: (Talk 1) Integral canonical models for Shimura varieties of Hodge type

Abstract: I will give an overview of the construction of integral models for Shimura varieties of Hodge type via the methods of Kisin and Faltings (as well as Vasiu). Since these Shimura varieties have no moduli interpretation, the methods are necessarily ad hoc, involving Deligne's theory of absolute Hodge cycles, p -adic Hodge theory and the deformation theory for p -divisible groups. I will quickly review these, and indicate the key difficulties in the construction.

Title: (Talk 2) Regular models for Shimura varieties of orthogonal type and the Tate conjecture for $K3$ surfaces in odd characteristic

Abstract: I will show how to construct good integral models for orthogonal Shimura varieties even at (odd) primes where the discriminant is not invertible. Then we will see that the certain moduli spaces of polarized $K3$ surfaces can be viewed as open sub-schemes of these integral models. This, combined with recent results of Kisin towards the Langlands-Rapoport conjecture, will lead us to a proof of the Tate conjecture in odd characteristic p for $K3$ surfaces admitting a polarization of degree indivisible by p^2 . The same methods also prove the Tate conjecture for cubic fourfolds in odd characteristic.

Speaker: Yuya Matsumoto (Tokyo)

Title: Good reduction criterion for $K3$ surfaces

Abstract: Abelian varieties over local fields have good reduction if and only if their l -adic étale cohomology (equivalently Tate modules) are unramified [Serre–Tate]. I prove a similar result for (potential) good reduction of projective $K3$ surfaces of small degree (relative to the residue characteristic). This result covers wider classes of $K3$ surfaces than those in my previous talks (in Kyoto, Hiroshima, Kyushu).

Speaker: Yoichi Mieda (Kyoto/Math Dept & Hakubi center)

Title: Potentially good reduction loci of open Shimura varieties and their l -adic cohomology

Abstract: If a Shimura variety is compact and has a proper integral model, its l -adic cohomology can be computed by the nearby cycle complex. However, if it is not compact, we do not have such a result in general. In this talk, for a certain class of Shimura varieties, I will introduce rigid open subsets called the potentially good reduction loci. If a Shimura variety has a reasonable integral model, then the l -adic cohomology of this open subset will coincide with the cohomology of the nearby cycle complex. We compare the compactly supported l -adic cohomology of a Shimura variety and that of its potentially good reduction locus. This is a joint work with Naoki Imai.

Speaker: Kentaro Nakamura (Hokkaido)

Title: A generalization of Kato’s local ε conjecture for (φ, Γ) -modules over the Robba ring

Abstract: In his preprint “Lectures on the approach to Iwasawa theory of Hasse-Weil L-functions via B_{dR} , Part II”, Kazuya Kato proposed a conjecture which is called local ε conjecture. This conjecture roughly says that the determinant of the Galois cohomology of a family of p -adic Galois representations has a canonical base which can be described by using Deligne-Langlands ε constants. In our talk, we generalize his conjecture for families of (φ, Γ) -modules over the Robba ring and prove a part of this conjecture in some special cases (more precisely, trianguline case). The two key ingredients are the recent result of Kedlaya-Pottharst-Xiao on the finiteness of cohomologies of (φ, Γ) -modules and my result on Bloch-Kato exponential maps for (φ, Γ) -modules. This talk is a work in progress.

Speaker: Benjamin Schraen (CNRS/UVSQ)

Title: (Talk 1) Completed cohomology of the tower of modular curves and local-global compatibility

Abstract: We will introduce completed p -adic étale cohomology of the tower of modular curves. We will next explain the strategy of Emerton to realize the local p -adic Langlands correspondence for $\text{GL}_2(\mathbb{Q}_p)$ in this cohomology.

Title: (Talk 2) The cohomology of local systems on quotient of Drinfel’d upper half spaces

Abstract: (joint work with Jan Kohlhaase) If X is a d -dimensional variety uniformized by

Drinfel'd upper half space, Schneider has conjectured the transversality of two natural filtrations on the de Rham cohomology of local systems on X . We will explain how vanishing theorems for the cohomology of discrete groups on locally analytic representations of GL_{d+1} could be used to prove this conjecture, and effectively prove any new cases.

Speaker: Xu Shen (Orsay)

Title: Cell decomposition and Lefschetz trace formula for some unitary group Rapoport-Zink spaces

Abstract: We explain that, how to use Fargues's theory of Harder-Narasimhan filtration for finite flat group schemes to find certain compact analytic domain in the basic Rapoport-Zink spaces of signature $(1, n - 1)$ for some unitary groups, so that its translations under the two related p -adic reductive groups form a locally finite covering of the whole space. By studying the action of regular elliptic elements and applying Mieda's theorem, we can deduce a Lefschetz trace formula for all large enough analytic domains coming from this covering. This will be sufficient for applications to representation theory.

Speaker: Sug Woo Shin (MIT)

Title: Integral Manin problem with PEL structure

Abstract: The rational/integral Manin problem asks whether a given p -divisible group over the algebraic closure of \mathbb{F}_p can be realized as the p -divisible group of an abelian variety over the same field up to an isogeny/isomorphism. The analogous problem for p -divisible groups and abelian varieties with PEL structure have been solved by Vasiu and Viehmann-Wedhorn for p -unramified PEL data of type A and C. I will explain a different approach based on Honda-Tate theory and Galois cohomology to obtain the answer without any unramified hypothesis. As an application one obtains a relatively simple argument to show the non-emptiness of Newton strata of Shimura varieties attached to p -unramified PEL data of type A and C.

Speaker: Takahiro Tsushima (Kyushu)

Title: Conjecture on the stable reduction of the Lubin-Tate curve and the Lusztig theory over finite rings

Abstract: A connection between the non-abelian Lubin-Tate theory and the Deligne-Lusztig theory seems to be mysterious. T. Yoshida proves that the Deligne-Lusztig variety for GL_n appears in the stable reduction of the Lubin-Tate space of level 1.

In this talk, we construct a curve over a finite field with some group action using Lusztig theory over finite rings and study its cohomology. We state a conjecture that this curve appears in the stable reduction of the Lubin-Tate curve. As an evidence, we prove this conjecture in the case where the level is two. This is a joint work with Tetsushi Ito and Yoichi Mieda.