

Workshop on the arithmetic geometry of Shimura varieties and Rapoport-Zink spaces

以下の通り研究集会を行います。皆様のご参加をお待ちしております。

日程 2011年7月4日(月)～8日(金)
場所 京都大学理学部3号館 110教室
世話人 伊藤哲史(京都大学数学教室)
ホームページ <http://www.math.kyoto-u.ac.jp/~tetsushi/workshop201107/index.html>

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プログラム

7月4日(月)

- 10:30–10:45 Registration
- 11:00–12:00 Tetsushi Ito (Kyoto, Math Dept)
Shimura varieties and Rapoport-Zink spaces — an introduction
- 14:00–15:00 Kentaro Nakamura (Keio)
Zariski density of crystalline representations for any p -adic field
- 15:30–17:00 Chung Pang Mok (McMaster)
Galois representations associated to automorphic forms of GL_2 over CM fields and local-global compatibility up to semi-simplification

7月5日(火)

- 10:00–11:00 Noriyuki Abe (Hokkaido)
On a classification of irreducible admissible modulo p representations of a split p -adic group
- 11:15–12:15 Yoichi Mieda (Kyushu)
Lefschetz trace formula and ℓ -adic cohomology of the Rapoport-Zink spaces for $GSp(4)$
- 14:00–15:00 Ana Caraiani (Harvard)
Local-global compatibility and monodromy
- 15:30–16:30 Dong Uk Lee (POSTECH)
Decomposition of Shimura datum of Hodge type
- 19:00– Banquet

7月6日 (水)

10:00–11:00 Yuya Matsumoto (Tokyo)

On good reduction of some $K3$ surfaces

11:15–12:15 Sug Woo Shin (MIT/KIAS)

Supercuspidal part of the mod ℓ cohomology of compact $GU(1, n - 1)$ Shimura varieties

(Free Afternoon)

7月7日 (木)

10:00–11:00 Naoki Imai (Kyoto, RIMS)

Cohomology of rigid curves with semi-stable coverings

11:15–12:15 Takahiro Tsushima (Kyushu)

On the elementary computation of the stable reduction of $\mathcal{X}(\pi^2)$ and its cohomology

14:00–15:00 Przemyslaw Chojecki (Ecole Polytechnique)

Density of crystalline points and local-global compatibility of Emerton

15:30–16:30 Tetsushi Ito (Kyoto, Math Dept)

ℓ -adic cohomology of Rapoport-Zink spaces and supercuspidal packets

7月8日 (金)

10:00–11:00 Miaofen Chen (Paris-Sud)

The determinant morphism for Rapoport-Zink spaces

11:15–12:15 Shushi Harashita (Yokohama National Univ)

Deligne-Lusztig varieties associated to G_2

14:00–15:00 Zhiwei Yun (MIT)

Arithmetic Fundamental Lemma for function fields

15:30–16:30 Kai-Wen Lan (Princeton)

Kodaira-type vanishing on good reduction fibers of Shimura varieties

アブストラクト

講演者: Noriyuki Abe (Hokkaido)

講演題目: On a classification of irreducible admissible modulo p representations of a split p -adic group

アブストラクト: In the theory of modulo p representations of a split p -adic group, the notion of a supersingular representation is defined using the Satake transform. I give a classification of irreducible admissible modulo p representations in terms of supersingular representations. In particular, it is proved that an irreducible admissible representation is supersingular if and only if it is supercuspidal. These are generalizations of results of Barthel-Livne and Herzig.

講演者: Ana Caraiani (Harvard)

講演題目: Local-global compatibility and monodromy

アブストラクト: Given a cuspidal automorphic representation of $GL(n)$ over a CM field which is regular algebraic and conjugate self-dual, one can associate to it a Galois representation. This Galois representation is known in most cases to be compatible with local Langlands. When n is even, the compatibility is known up to semisimplification or when the representation satisfies an additional regularity condition. I will extend the compatibility to Frobenius semisimplification by identifying the monodromy operators on either side.

講演者: Miaofen Chen (Paris-Sud)

講演題目: The determinant morphism for Rapoport-Zink spaces

アブストラクト: We will prove a local analogue on the nonarchimedean places of the determinant morphism for Shimura varieties defined by Deligne. Let \mathcal{M} be an unramified Rapoport-Zink space of EL type or PEL type which is unitary or symplectic. Let \mathcal{M}_{rig} be the generic fiber of Berthelot of \mathcal{M} in the sense of rigid analytic spaces over which there exists a tower of rigid analytic spaces $(\mathcal{M}_K)_K$ classifying the level structures. The objective of the talk is to study the geometrically connected components of this tower. In order to do this, we define a determinant morphism \det_K from the tower $(\mathcal{M}_K)_K$ to a tower of rigid analytic spaces of dimension 0 associated to the cocenter of the reductive group related to the space \mathcal{M} . We prove that the geometric fibers of the determinant morphism \det_K are the geometrically connected components of \mathcal{M}_K .

講演者: Przemyslaw Chojacki (Ecole Polytechnique)

講演題目: Density of crystalline points and local-global compatibility of Emerton

アブストラクト: We will prove that (almost) crystalline points are dense inside the completed

cohomology of unitary Shimura varieties of Harris-Taylor. Then, we will explain how this result leads to the weak version of local-global compatibility of Emerton for Shimura curves over totally real fields where p is totally decomposed. Assuming certain modularity conjecture, this will permit us to deduce the Fontaine-Mazur conjecture in this case.

講演者: Shushi Harashita (Yokohama National Univ)

講演題目: Deligne-Lusztig varieties associated to G_2

アブストラクト: In some Shimura varieties, Deligne-Lusztig varieties appear as natural subvarieties. It would be important to understand the geometry of Deligne-Lusztig varieties. In this lecture, I am going to talk about defining equations and the affineness of (fine generalized) Deligne-Lusztig varieties associated to split G_2 .

講演者: Naoki Imai (Kyoto, RIMS)

講演題目: Cohomology of rigid curves with semi-stable coverings

アブストラクト: In this talk, we will explain construction of a semi-stable formal model of a rigid curve with a semi-stable covering. Then we consider a finite flat morphism between two rigid curves with semi-stable coverings. We discuss the induced pullback and pushforward between the ℓ -adic cohomology of the rigid curves. This is joint work with Takahiro Tsushima.

講演者: Tetsushi Ito (Kyoto)

講演題目: Shimura varieties and Rapoport-Zink spaces — an introduction

アブストラクト: The aim of this talk is to motivate the participants of the workshop to work on Shimura varieties and Rapoport-Zink spaces. I will give a brief introduction to Shimura varieties and Rapoport-Zink spaces with a special emphasis on geometry and cohomology. I will also discuss several applications to the local/global Langlands correspondences and Galois representations. Many important results have been obtained so far, but so many challenging problems are still open.

講演題目: Cohomology of Rapoport-Zink spaces and supercuspidal packets

アブストラクト: The ℓ -adic cohomology of Rapoport-Zink spaces is expected to realize local Langlands and local Jacquet-Langlands correspondences. Supercuspidal representations are the “most interesting” ones, and they tend to concentrate in the middle degree cohomology. However, for the Rapoport-Zink space of $\mathrm{GSp}(4, \mathbb{Q}_p)$, in a recent joint work with Yoichi Mieda, we discovered certain supercuspidal representations do appear outside the middle degree. This phenomenon is related to A -packets of Saito-Kurokawa representations of $\mathrm{GSp}(4, \mathbb{A})$. In this talk, I try to discuss a conjectural description of the degree of cohomology of supercuspidal representations in terms of

the structure of local L & A -packets and Zelevinsky duality. This might be considered as a local analogue of Arthur's conjecture on the classification of L^2 automorphic representations.

講演者: Kai-Wen Lan (Princeton)

講演題目: Kodaira-type vanishing on good reduction fibers of Shimura varieties

アブストラクト: While the classical Kodaira vanishing theorem is false in general in positive characteristics, Deligne, Illusie and Raynaud showed that it remains true under some mild (liftability and dimension) conditions. Since then it has been generalized in two directions: Esnault and Viehweg allowed the line bundle to be somewhat less than ample, and Illusie allowed certain nontrivial coefficients of geometric origin. I will explain a common generalization of these two, which implies interesting vanishing results for the cohomology of (noncompact) PEL-type Shimura varieties with torsion automorphic coefficients. (This is joint work with Junecue Suh.)

講演者: Dong Uk Lee (POSTECH)

講演題目: Decomposition of Shimura datum of Hodge type

アブストラクト: We prove that arbitrary Shimura datum of Hodge type admits a covering into a product of certain abelian type Shimura data associated with its adjoint datum.

講演者: Yuya Matsumoto (Tokyo)

講演題目: On good reduction of some $K3$ surfaces

アブストラクト: Let X be a variety over a local field K . If X is an abelian variety, a theorem of Serre-Tate shows that X has good reduction if and only if its ℓ -adic étale cohomology is unramified (a Galois representation of K is unramified if the action of the inertia group is trivial). In this talk, I prove that similar results hold if X belongs to certain classes of $K3$ surfaces.

講演者: Yoichi Mieda (Kyushu)

講演題目: Lefschetz trace formula and ℓ -adic cohomology of the Rapoport-Zink spaces for $\mathrm{GSp}(4)$

アブストラクト: Rapoport-Zink spaces are certain moduli spaces of quasi-isogenies of p -divisible groups with additional structures and can be regarded as local analogues of Shimura varieties. Kottwitz' conjecture predicts that their ℓ -adic cohomology partially realize the local Langlands correspondence and the local Jacquet-Langlands correspondence for rather general reductive groups. In this talk, we will investigate the ℓ -adic cohomology of the Rapoport-Zink space for $\mathrm{GSp}(4)$ by using Lefschetz trace formula for open adic spaces, and give a partial result on the conjecture of Kottwitz.

講演者: Chung Pang Mok (McMaster)

講演題目: Galois representations associated to automorphic forms of GL_2 over CM fields and local-global compatibility up to semi-simplification

アブストラクト: Let E be a CM extension over a totally real field F . Given a regular algebraic cuspidal automorphic representation π on $GL_2(E)$, whose central character is invariant under Galois conjugation of E/F , we construct the corresponding compatible system of 2-dimensional p -adic Galois representations attached to π . The basic idea, originally due to Harris-Soudry-Taylor, is to consider lifting to $GSp_4(F)$. We present two constructions: one originally due to Taylor, by using p -power congruences; another one depends on using eigenvariety for $GSp_4(F)$ (recently constructed by Vincent Pilloni and others). Time allowed we will also indicate the proof of local-global compatibility up to semi-simplification. This whole work was made possible by Arthur's classification of discrete spectrum of GSp_4 .

講演者: Kentaro Nakamura (Keio)

講演題目: Zariski density of crystalline representations for any p -adic field

アブストラクト: I would like to talk about a theorem concerning Zariski density of n -dimensional crystalline representations for any n and for any p -adic field K . This theorem is the generalization of Colmez and Kisin's theorem in the case $n = 2$ and $K = \mathbb{Q}_p$, which played many crucial roles in their studies of p -adic local Langlands for $GL_2(\mathbb{Q}_p)$. The key for the proof is to construct a p -adic family of trianguline representations. We construct this family by generalizing Kisin's theory of finite slope subspace X_{fs} . We note that Chenevier recently proved this theorem for any n and for $K = \mathbb{Q}_p$ by generalizing Colmez's proof.

講演者: Sug Woo Shin (MIT/KIAS)

講演題目: Supercuspidal part of the mod ℓ cohomology of compact $GU(1, n-1)$ Shimura varieties

アブストラクト: Let p be a prime where $GU(1, n-1)$ becomes isomorphic to a general linear group. We describe the mod ℓ cohomology of the Shimura varieties in the part where $G(\mathbb{Q}_p)$ acts by a mod ℓ supercuspidal representation. Any prime ℓ and an arbitrarily deep level structure at ℓ are allowed. The method is different from that of Emerton and Gee on a similar problem, and the results seem to be complementing each other.

講演者: Takahiro Tsushima (Kyushu)

講演題目: On the elementary computation of the stable reduction of $\mathcal{X}(\pi^2)$ and its cohomology

アブストラクト: Let F be a local field with integer ring \mathcal{O}_F , π a uniformizer and k the residue

field. The Lubin-Tate space $\mathcal{X}(\pi^n)$ is a deformation space of the unique (up to isomorphism) formal \mathcal{O}_F -module Σ/\bar{k} equipped with Drinfeld level π^n -structure. Recently, Jared Weinstein finds a systematic way to find components in the stable reduction of the Lubin-Tate tower, using p -adic Hodge theory, etc. In my talk, on the basis of the work of R. Coleman and K. McMurdy on the stable reduction of the modular curve $X_0(p^3)$, we explicitly calculate the stable reduction of $\mathcal{X}(\pi^2)$. Our method is very elementary only using blow-ups. As a result of the computation, we explicitly determine the action of GL_2 , the division algebra with invariant $1/2$, and the inertia group on the stable reduction. As an application of these computations, we observe that the local Jacquet-Langlands correspondence and the local Langlands correspondence for GL_2 , for the supercuspidal representations of level 1 or $1/2$, are realized in the étale cohomology H^1 of the Lubin-Tate space $\mathcal{X}(\pi^2)$. We also discuss a relationship with the cohomology H_c^2 of Lusztig surface (affine) for $\mathrm{GL}_2(\mathcal{O}_F/\pi^2)$. This work is a joint work with Yoichi Mieda and Tetsushi Ito.

講演者: Zhiwei Yun (MIT)

講演題目: Arithmetic Fundamental Lemma for function fields

アブストラクト: Arithmetic Fundamental Lemma (AFL) is a conjecture of Wei Zhang relating orbital integrals of $\mathrm{GL}(n)$ and intersection numbers on the unitary Rapoport-Zink spaces. We will sketch a proof of this conjecture in the function field case. We will first establish a global analog of AFL on counting certain Shtukas, and then deduce the AFL from its global analog.