

# ミニ研究集会「数論幾何とその周辺」

下記の通り研究集会を行います。皆様のご参加をお待ちしています。

日程	2010年7月19日（月，祝日（海の日））
場所	京都大学理学研究科数学教室 理学部3号館108教室
講演者	加藤和也（シカゴ大学），Luc Illusie（パリ南大学/東京大学）， 臼井三平（大阪大学），Mohamed Saidi（エクセター大学/京大数理研）， Anna Cadoret（I.M.B. - ボルドー第1大学）

## プログラム

### 7月19日（月，祝日（海の日））

- 10:00～11:00 加藤和也（シカゴ大学）  
Moduli spaces of log mixed Hodge structures and application to  
construction of Néron models, I  
(joint work of K. Kato, C. Nakayama, and S. Usui)
- 11:10～12:10 Luc Illusie（パリ南大学/東京大学）  
Independence of families of  $\ell$ -adic representations, after J-P. Serre
- 14:00～15:00 臼井三平（大阪大学）  
Moduli spaces of log mixed Hodge structures and application to  
construction of Néron models, II  
(joint work of K. Kato, C. Nakayama, and S. Usui)
- 15:30～16:30 Mohamed Saidi（エクセター大学/京大数理研）  
Fake liftings of Galois covers between smooth curves
- 16:40～17:40 Anna Cadoret（I.M.B. - ボルドー第1大学）  
On the Galois module structure of the generic  $\ell$ -torsion of  
an abelian scheme (joint work with Akio Tamagawa)

※ 講演のアブストラクトや研究集会に関する最新の情報はホームページ  
<http://www.math.kyoto-u.ac.jp/~tetsushi/workshop20100719/indexj.html>  
をご覧ください。

世話人・連絡先： 玉川安騎男（京大数理研）  
伊藤哲史（京大数学教室）

## タイトル・アブストラクト

**講演者:** I. 加藤和也 (シカゴ大学) & II. 臼井三平 (大阪大学)

**講演題目:** Moduli spaces of log mixed Hodge structures and application to construction of Néron models, I, II (joint work of K. Kato, C. Nakayama, and S. Usui)

**アブストラクト:** We construct toroidal partial compactifications of the moduli spaces of mixed Hodge structures with polarized graded quotients. They are fine moduli spaces of log mixed Hodge structures with polarized graded quotients. We also apply them to construct Néron models of intermediate Jacobians over higher dimensional bases.

**講演者:** Mohamed Saidi (エクセター大学/京大数理研)

**講演題目:** Fake liftings of Galois covers between smooth curves

**アブストラクト:** We introduce the notion of fake liftings of cyclic covers between smooth curves, which only exist if the Oort conjecture on liftings of cyclic covers between smooth curves is false, and establish some of their basic properties.

**講演者:** Luc Illusie (パリ南大学/東京大学)

**講演題目:** Independence of families of  $\ell$ -adic representations, after J-P. Serre

**アブストラクト:** Let  $k$  be a number field,  $\bar{k}$  an algebraic closure of  $k$ ,  $\Gamma_k = \text{Gal}(\bar{k}/k)$ . A family of continuous homomorphisms  $\rho_\ell : \Gamma_k \rightarrow G_\ell$ , indexed by prime numbers  $\ell$ , where  $G_\ell$  is a locally compact  $\ell$ -adic Lie group, is said to be independent if  $\rho(\Gamma_k) = \prod \rho_\ell(\Gamma_k)$ , where  $\rho = (\rho_\ell) : \Gamma_k \rightarrow \prod G_\ell$ . Serre gave a criterion for such a family to become independent after a finite extension of  $k$ . I will explain Serre's criterion and show that it applies to families coming from the  $\ell$ -adic cohomology (or cohomology with compact support) of schemes separated and of finite type over  $k$ .

**講演者:** Anna Cadoret (I.M.B. - ボルドー第1大学)

**講演題目:** On the Galois module structure of the generic  $\ell$ -torsion of an abelian scheme (joint work with Akio Tamagawa)

**アブストラクト:** Let  $k$  be a field of characteristic 0,  $S$  a smooth, separated, geometrically connected curve over  $k$  with generic point  $\eta$  and  $A \rightarrow S$  an abelian scheme. Let  $\pi_1(S)$  denote the étale fundamental group of  $S$ . Then, for each prime  $\ell$ , one gets the natural representation:

$$\rho_\ell : \pi_1(S) \rightarrow \text{GL}(A_\eta[\ell]).$$

Via the theory of étale fundamental groups, one can associate to this representation curves which are natural generalizations of the classical modular curves  $Y(\ell)$ ,  $Y_1(\ell)$  and which we call abstract modular curves.

The structure of  $A_\eta[\ell]$  as a  $\pi_1(S)$ -module encodes lots of information about abstract modular curves. A key result is the following. Given a  $\pi_1(S)$ -submodule  $M \subset A_\eta[\ell]$ , write  $\rho_M : \pi_1(S) \rightarrow \text{GL}(M)$  for the induced representation and set  $G_M := \rho_M(\pi_1(S))$ . Then:

- (1)  $A_\eta[\ell]$  is a semisimple  $\pi_1(S)$ -module,  $\ell \gg 0$ .
- (2) There exists an integer  $B = B(A) \geq 1$  such that for any prime  $\ell$ ,  $\pi_1(S)$ -submodule  $M \subset A_\eta[\ell]$  and any abelian normal subgroup  $C \triangleleft G_M$ ,  $|C| \leq B$ .

I will sketch the proof of this statements and, if I have time, explain how to derive from it estimates for the genus and gonality of abstract modular curves when  $\ell \gg 0$ .