Mini-workshop on Arithmetic Geometry and Related Topics

Date July 19th (Mon, Holiday), 2010

Location Department of Mathematics, Kyoto University
Falucty of Science Bldg No.3, Lecture Room 108

Speakers Kazuya Kato (Chicago), Luc Illusie (Paris-Sud/Tokyo),
Sampei Usui (Osaka), Mohamed Saidi (Exeter/Kyoto RIMS),
Anna Cadoret (I.M.B. - Univ. Bordeaux 1)

Program

July 19th (Mon, Holiday)

10:00〜11:00 Kazuya Kato (Chicago)
Moduli spaces of log mixed Hodge structures and application to construction of Néron models, I
(joint work of K. Kato, C. Nakayama, and S. Usui)

11:10〜12:10 Luc Illusie (Paris-Sud/Tokyo)
Independence of families of ℓ-adic representations, after J-P. Serre

14:00〜15:00 Sampei Usui (Osaka)
Moduli spaces of log mixed Hodge structures and application to construction of Néron models, II
(joint work of K. Kato, C. Nakayama, and S. Usui)

15:30〜16:30 Mohamed Saidi (Exeter/Kyoto RIMS)
Fake liftings of Galois covers between smooth curves

16:40〜17:40 Anna Cadoret (I.M.B. - Univ. Bordeaux 1)
On the Galois module structure of the generic ℓ-torsion of an abelian scheme (joint work with Akio Tamagawa)

For details, please see the webpage of the workshop:

http://www.math.kyoto-u.ac.jp/~tetsushi/workshop20100719/index.html

Organizers: Akio Tamagawa (Kyoto, RIMS)
Tetsushi Ito (Kyoto, Dept. of Math.)

Titles & Abstracts

Speakers: I. Kazuya Kato (Chicago) & II. Sampei Usui (Osaka)

Title: Moduli spaces of log mixed Hodge structures and application to construction of Néron models, I, II (joint work of K. Kato, C. Nakayama, and S. Usui)

Abstract: We construct toroidal partial compactifications of the moduli spaces of mixed Hodge structures with polarized graded quotients. They are fine moduli spaces of log mixed Hodge structures with polarized graded quotients. We also apply them to construct Néron models of intermediate Jacobians over higher dimensional bases.

Speaker: Mohamed Saidi (Exeter/Kyoto RIMS)

Title: Fake liftings of Galois covers between smooth curves

Abstract: We introduce the notion of fake liftings of cyclic covers between smooth curves, which only exist if the Oort conjecture on liftings of cyclic covers between smooth curves is false, and establish some of their basic properties.

Speaker: Luc Illusie (Paris-Sud/Tokyo)

Title: Independence of families of ℓ-adic representations, after J-P. Serre

Abstract: Let k be a number field, $\overline{k}$ an algebraic closure of k, $\Gamma_k = \mathrm{Gal}(\overline{k}/k)$. A family of continuous homomorphisms $\rho_\ell : \Gamma_k \to G_\ell$, indexed by prime numbers ℓ, where G_ℓ is a locally compact ℓ-adic Lie group, is said to be independent if $\rho(\Gamma_k) = \prod \rho_\ell(\Gamma_k)$, where $\rho = (\rho_\ell) : \Gamma_k \to \prod G_\ell$. Serre gave a criterion for such a family to become independent after a finite extension of k. I will explain Serre's criterion and show that it applies to families coming from the ℓ-adic cohomology (or cohomology with compact support) of schemes separated and of finite type over k.

Speaker: Anna Cadoret (I.M.B. - Univ. Bordeaux 1)

Title: On the Galois module structure of the generic ℓ-torsion of an abelian scheme (joint work with Akio Tamagawa)

Abstract: Let k be a field of characteristic 0, S a smooth, separated, geometrically connected curve over k with generic point η and $A \to S$ an abelian scheme. Let $\pi_1(S)$ denote the étale fundamental group of S. Then, for each prime ℓ, one gets the natural representation:

$$\rho_\ell : \pi_1(S) \to \mathrm{GL}(A_\eta[\ell]).$$

Via the theory of étale fundamental groups, one can associate to this representation curves which are natural generalizations of the classical modular curves $Y(\ell)$, $Y_1(\ell)$ and which we call abstract modular curves.

The structure of $A_\eta[\ell]$ as a $\pi_1(S)$-module encodes lots of information about abstract modular curves. A key result is the following. Given a $\pi_1(S)$-submodule $M \subset A_\eta[\ell]$, write $\rho_M : \pi_1(S) \to \mathrm{GL}(M)$ for the induced representation and set $G_M := \rho_M(\pi_1(S))$. Then:

(1) $A_\eta[\ell]$ is a semisimple $\pi_1(S)$-module, $\ell \gg 0$.

(2) There exists an integer $B = B(A) \geq 1$ such that for any prime ℓ, $\pi_1(S)$-submodule $M \subset A_\eta[\ell]$ and any abelian normal subgroup $C \lhd G_M$, $|C| \leq B$.

I will sketch the proof of this statements and, if I have time, explain how to derive from it estimates for the genus and gonality of abstract modular curves when $\ell \gg 0$.