The Spectral Link in Turbulent Frictional Drag and Turbulent Mean Velocity Profile

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There is a missing link between the macroscopic properties of turbulent flows, such as the frictional drag of a wall-bounded flow, and the turbulent spectrum. I shall describe this missing "spectral link" as a new area of research of both scientific and practical importance. First, I shall outline a spectral theory of the frictional drag that expresses the frictional drag f as a functional of the turbulent spectrum. For the "energy cascade" spectrum, where the spectral exponent $\alpha = 5/3$, I shall obtain an analytical version of the arch-famous Nikuradse's diagram that is in minute qualitative agreement with the distinctive features diagram that have remained elusive to any theoretical elucidation. I shall show that the exponents of the empirical scalings in the diagram are are set by the spectral exponent. Next, I shall describe unprecedented experimental measurements of the frictional drag in turbulent soap-film flows over smooth walls, with two disparate types of turbulent spectra: the "enstrophy cascade," for which $\alpha = 3$, and the "inverse energy cascade," for which $\alpha = 5/3$. I shall show that the functional relation between f and Re is set by the spectral exponent α as per the predictions of the spectral theory of frictional drag: where $\alpha = 3$, $f \sim Re^{-1/2}$; where $\alpha = 5/3$, $f \sim Re^{-1/4}$. Last, I shall outline an analysis that yields the turbulent mean-velocity profile (MVP) of a pipe flow as a functional of the spatially varying turbulent spectrum. Each part of the MVP is a manifestation of a specific spectral range.