Strong approximation of Dyson's Brownian motion without boundary hitting

Dai Taguchi (Kansai University)
joint work with
Yushi Hamaguchi (Kyoto University)

Abstract

In this talk, we discuss strong convergence rates of the backward Euler–Maruyama scheme for Dyson's Brownian motion

$$\mathrm{d}X_i(t) = \sum_{j\neq i} \frac{k}{X_i(t) - X_j(t)}\,\mathrm{d}t + \mathrm{d}B_i(t),\ i = 1, \dots, d,\ X(0) = x_0 \in \mathbb{W}.$$

We deal with the full non-colliding regime $k \geq 1/2$, under which the process $X = (X(t))_{t\in[0,T]}$ never hits the boundary of the Weyl chamber $\mathbb{W} = \{x \in \mathbb{R}^d \,|\, x_1 > x_2 > \cdots > x_d\}$.

To handle the singular drift and preserve the chamber $\mathbb{W}$, we study the backward Euler–Maruyama scheme given by $X^{(n)}(0) := X(0)$ and

$$X_i^{(n)}(t_{\ell+1}) = X_i^{(n)}(t_\ell) + \sum_{j\neq i} \frac{k}{X_i^{(n)}(t_{\ell+1}) - X_j^{(n)}(t_{\ell+1})}\frac{T}{n} + B_i(t_{\ell+1}) - B_i(t_\ell),\ \ell = 0, 1, \dots, n-1,$$

where $t_\ell := \ell T/n$. This backward scheme is well-defined because, for any $x \in \mathbb{R}^d$, the nonlinear equation

$$y_i = x_i + \sum_{j\neq i} \frac{k}{y_i - y_j}\frac{T}{n},\ i = 1, \dots, d,$$

has a unique solution $y \in \mathbb{W}$.

We present explicit bounds for the L^2-supremum error

$$\mathbb{E}\Big[\sup_{\ell=1,\dots,n} |X(t_\ell) - X^{(n)}(t_\ell)|^2\Big]^{1/2}$$

for any $k \geq 1/2$. We will also introduce a Newton-type method with quadratic convergence to solve the nonlinear equation numerically.