LOWER BOUNDS FOR MEAN EXIT TIMES FOR LÉVY PROCESSES ON RIEMANNIAN MANIFOLDS

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Let (M, g) be a smooth, complete, and connected Riemannian manifold of dimension $m \geq 2$. We write $(O(M), \pi)$ for the orthonormal frame bundle of M with the canonical projection $\pi : O(M) \to M$. We regard each $u \in O(M)$ as the linear isomorphism from $\mathbb{R}^m$ to $T_{\pi(u)}M$. For $u \in O(M)$ and $z \in \mathbb{R}^m$, let $Hz(u) \in T_u O(M)$ denote the horizontal lift of $uz \in T_{\pi(u)}M$. Let $\{e_1, \ldots, e_m\}$ be the standard basis of $\mathbb{R}^m$, and set $H_i := He_i$ for $i = 1, \ldots, m$.

Let $\{(B_t^1, B_t^2, \cdots, B_t^m)\}_t$ be a standard m-dimensional Brownian motion. We write $\nu(dz)$ for the Lévy measure on $\mathbb{R}_0^m$. A Poisson random measure over $\mathbb{R}_0^m \times [0, \infty)$ with intensity measure $\hat{n}(dz, ds) = \nu(dz)ds$ is denoted by $N(dz, ds)$. The compensated Poisson random measure is defined by $\widetilde{N}(dz, ds) = N(dz, ds) - \hat{n}(dz, ds)$.

We consider the Marcus-type stochastic differential equation of the form:

$$\begin{aligned} dU_t = \kappa \sum_{i=1}^{m} H_i(U_{t-}) \circ dB_t^i + \int_{|z| \leq 1} \Big(\mathrm{Exp}(Hz)(U_{t-}) - U_{t-} \Big) \widetilde{N}(dz, dt) \\ + \int_{|z| > 1} \Big(\mathrm{Exp}(Hz)(U_{t-}) - U_{t-} \Big) N(dz, dt), \end{aligned}$$

where κ is a nonnegative constant. According to Kunita [13, Theorem7.1.1], this stochastic differential equation admits a unique càdlàg solution.

For a Borel set $B \subset M$, let

$$\sigma(B) := \inf\{t > 0 \mid U_t \notin \pi^{-1}(B)\}.$$

Here, $\{U_t \mid 0 \leq t < e\}$ is a solution to the Marcus-type stochastic differential equation.

In this talk, we consider lower bounds for the mean exit time $\mathbb{E}_u[\sigma(B(x, R))]$, where $x \in M$, $R > 0$, and $u \in O(M)$. The lower bounds are expressed as follows: Let $\Phi : (0, \infty) \to \mathbb{R}$ be the function determined by κ and ν. For given $x \in M$ and $R > 0$, let $S^*(\cdot, B(x, R)) : M \to \mathbb{R}$ be the function determined by the geometry of $B(x, R) \subset M$. Under suitable assumptions, we have

$$\mathbb{E}_u[\sigma(B(x, R))] \geq \Phi(R) \cdot S^*(\pi(u), B(x, R))$$

for all $x \in M$, $R > 0$ and $u \in O(M)$. As an application, we derive an explicit lower bound when the cut locus of M is empty and its sectional curvature is bounded from below.

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