

Strong uniqueness of Dirichlet operators related to stochastic quantization for the $\exp(\Phi)_2$ -model

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This talk is based on a joint work with Hiroshi Kawabi (Keio University). Let $\Lambda = (\mathbb{R}/2\pi\mathbb{Z})^2$ be the two dimensional torus and $H^s(\Lambda)$, $s \in \mathbb{R}$ denote the L^2 -Sobolev space of order s with periodic boundary condition. We put $H := L^2(\Lambda)$. Let μ_0 be the mean-zero Gaussian measure on $\mathcal{D}'(\Lambda)$, with the covariance operator $(1 - \Delta)^{-1}$. We define the $\exp(\Phi)_2$ -measure $\mu^{(\alpha)}$ by

$$\mu^{(\alpha)}(d\phi) = \frac{1}{Z^{(\alpha)}} \exp \left(- \int_{\Lambda} \llbracket \exp(\alpha\phi) \rrbracket(x) dx \right) \mu_0(d\phi),$$

where $\alpha \in (-\sqrt{8\pi}, \sqrt{8\pi})$ is the charge constant, $Z^{(\alpha)} > 0$ is the normalizing constant and the Wick exponential is formally introduced by the expression

$$\llbracket \exp(\alpha\phi) \rrbracket(x) = \exp \left(\alpha\phi(x) - \frac{\alpha^2}{2} \mathbb{E}^{\mu_0} [\phi(x)^2] \right), \quad x \in \Lambda.$$

Note that the diverging term $\mathbb{E}^{\mu_0} [\phi(x)^2]$ plays a role of the Wick renormalization.

We now set $E := H^{-\beta}(\Lambda)$ for a suitable constant $\beta \in (0, 1)$ and consider a pre-Dirichlet form $(\mathcal{E}, \mathfrak{F}C_b^\infty)$ defined by

$$\mathcal{E}(F, G) = \frac{1}{2} \int_E (D_H F(\phi), D_H G(\phi))_H \mu^{(\alpha)}(d\phi), \quad F, G \in \mathfrak{F}C_b^\infty,$$

where $\mathfrak{F}C_b^\infty$ is the set of all smooth cylindrical functions on E and D_H denotes the H -derivative. Applying the integration by parts formula with respect to $\mu^{(\alpha)}$, we have

$$\mathcal{E}(F, G) = - \int_E \mathcal{L}F(\phi) G(\phi) \mu^{(\alpha)}(d\phi), \quad F, G \in \mathfrak{F}C_b^\infty,$$

where

$$\mathcal{L}F(\phi) = \frac{1}{2} \text{Tr}(D_H^2 F(\phi)) - \frac{1}{2} \langle \phi, (1 - \Delta) D_H F(\phi) \rangle - \frac{\alpha}{2} \langle \llbracket \exp(\alpha\phi) \rrbracket, D_H F \rangle, \quad \phi \in E,$$

and $\mathcal{L}F \in L^p(\mu^{(\alpha)})$ for all $\alpha^2 < 8\pi$ and $p \geq 1$. This means that the pre-Dirichlet operator $(\mathcal{L}, \mathfrak{F}C_b^\infty)$ is dissipative (and hence closable) in $L^p(\mu^{(\alpha)})$.

Our main result in this talk is the following.

Theorem. *Let $p \geq 2$ and the charge constant α satisfies*

$$\alpha^2 < \frac{16\pi}{9p - 6}.$$

Then the pre-Dirichlet operator $(\mathcal{L}, \mathfrak{F}C_b^\infty)$ is L^p -unique, that is, there exists exactly one C_0 -semigroup in $L^p(\mu^{(\alpha)})$ such that its generator extends $(\mathcal{L}, \mathfrak{F}C_b^\infty)$. In particular, $(\mathcal{L}, \mathfrak{F}C_b^\infty)$ is essentially self-adjoint in $L^2(\mu^{(\alpha)})$ provided that $\alpha^2 < 4\pi/3$.