A generalized coupling approach for the weak approximation of stochastic functional differential equations

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Abstract

In this talk, we consider a numerical approximation for stochastic functional differential equations:

$$\mathrm{d}X(t) = b(t, X)\,\mathrm{d}t + \sigma(t, X)\,\mathrm{d}W(t), \quad t \in [0, \infty),$$

on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where W is a d-dimensional Brownian motion, and the coefficients $b : [0, \infty) \times C([0, \infty); \mathbb{R}^n) \to \mathbb{R}^n$ and $\sigma : [0, \infty) \times C([0, \infty); \mathbb{R}^n) \to \mathbb{R}^{n \times d}$ are progressively measurable maps. Under the assumption that the coefficients are (locally) Hölder continuous and the diffusion coefficient satisfies (non-uniform) ellipticity condition, we provide an estimate for the Lévy–Prokhorov metric between the weak solution of stochastic functional differential equation and the corresponding Euler–Maruyama scheme. The idea of the proof is based on the "generalized coupling approach" which has been studied by Butkovsky–Kulik–Scheutzow [1] and Kulik–Scheutzow [2]. We apply our results to Markovian SDEs, for example Dyson's Brownian motion:

$$\mathrm{d}X_i(t) = \sum_{j \neq i} \frac{\mu}{X_i(t) - X_j(t)}\,\mathrm{d}t + \mathrm{d}W_i(t), \quad i = 1, \ldots, n,$$

non-Markovian SDEs, for example stochastic delay/integro differential equations:

$$\mathrm{d}X(t) = b(X(t-\tau), X(t))\,\mathrm{d}t + \sigma(X(t-\tau), X(t))\,\mathrm{d}W(t),$$
$$\mathrm{d}X(t) = b\Big(\int_0^t K(t-s)X(s)\,\mathrm{d}s\Big)\,\mathrm{d}t + \sigma\Big(\int_0^t K(t-s)X(s)\,\mathrm{d}s\Big)\,\mathrm{d}W(t),$$

reflected SDEs:

$$\mathrm{d}X(t) = b(t, X(t))\,\mathrm{d}t + \sigma(t, X(t))\,\mathrm{d}W(t) + \mathrm{d}\Phi(t),$$

and perturbed diffusion processes ($\alpha \in [0, 1)$):

$$X(t) = X(0) + \int_0^t b(s, X(s))\,\mathrm{d}s + \int_0^t \sigma(s, X(s))\,\mathrm{d}W(s) + \alpha \max_{s \in [0,t]} X(s).$$

References

[1] Butkovsky, O., Kulik, A. and Scheutzow, M. Generalized couplings and ergodic rates for SPDEs and other Markov models. *Ann. Appl. Probab.* **30**(1) 1–39 (2020).

[2] Kulik, A. and Scheutzow, M. Well-posedness, stability and sensitivities for stochastic delay equations: a generalized coupling approach. *Ann. Appl. Probab.* **48**(6) 3041–3076 (2020).