# On scale functions of standard processes with no positive jumps

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This talk is based on [5], which is a joint work with Kosuke Yamato (University of Tsukuba), and [4].

A 1-dimensional Lévy process that has no positive jumps and no monotone paths is called a spectrally negative Lévy process. In the theory of spectrally Lévy processes, the scale functions are known as a powerful tool for characterizing their behavior. These functions can represent the Laplace transform of the exit times from intervals of spectrally negative Lévy processes. Another important property is that the scale functions can characterize the potential densities of spectrally negative Lévy processes killed on exiting intervals. These properties allow us to obtain various results, such as the characterization of the economic cost at the moment of ruin in the risk theory, the derivation of optimal strategies in stochastic control problems, and the characterization of some types of invariant measures. For the above reasons, many researchers have attempted to define scale functions for other stochastic processes with no positive jumps and apply them. The contents presented in this talk is part of such research.

This presentation is as follows.

(i) With reference to [1] and [2], we recall the properties of the scale functions of spectrally negative Lévy processes and its applications.

(ii) In [3], I defined the scale functions of standard processes with no positive jumps using the excursion measures. We recall their definitions and properties.

(iii) Based on [5], we see an analytic result and a representation using Volterra integral equations of the scale functions defined in [3]. This is the first new result.

(iv) Based on [4], we give a representation using Volterra integral equations of the scale functions of space-time changed standard processes with no positive jumps. This is the second new result. For this proof, the result obtained in [5] is important.

Using this result, we can obtain a representation of the scale functions such as self-similar Markov processes with no positive jumps using Volterra integral equations.

## References

[1] A. Kuznetsov, A. E. Kyprianou, and V. Rivero. The theory of scale functions for spectrally negative Lévy processes. In *Lévy matters II*, Vol. 2061 of *Lecture Notes in Math.*, pp. 97–186. Springer, Heidelberg, 2012.

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