Generalization of the fourth moment theorem

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The fourth moment theorem by Nualart and Peccati [3] characterizes central convergence of multiple Wiener integrals by convergences of the second and the fourth moments of the integrals and is applicable to many problems (*e.g.* approximation theory of stochastic differential equations driven by fractional Brownian motions). One of natural questions arising from this theorem is whether we can characterize the central convergence by convergences of any two even moments of Wiener integrals. In this talk we consider this question along the same line with [1] and see results obtained in [2].

Here we state our main theorem. Let $X = \{X(h)\}_{h\in\mathfrak{H}}$ be an isonormal Gaussian random variables parametrized by a separable Hilbert space $\mathfrak{H}$. For an integer $p \geq 0$, $\mathcal{H}_p$ stands for the pth Wiener chaos, which is a set of multiple Wiener integrals of order p. Let $\{Z_n\}_{n=1}^{\infty} \subset \mathcal{H}_p$ for some $p \geq 2$ and N be a standard Gaussian random variable. Then we have the next theorem.

Theorem 1. *Let $\mathcal{I}$ be any of the following.*

(1) $\mathcal{I} = \{2, 2k\}$, *where $2k \geq 4$ is an arbitrary even integer.*

(2) $\mathcal{I} = \{4, 2k\}$, *where $2k \geq 6$ is an arbitrary even integer.*

(3) $\mathcal{I} = \{6, 8\}, \{6, 10\}$.

(4) $\mathcal{I} = \{6, 12, 14, 2k\}$, *where $2k \geq 16$ is an arbitrary even integer.*

(5) $\mathcal{I} = \{6, 12, 18, 30, 32, 2k\}$, *where $2k \geq 34$ is an arbitrary even integer.*

If it holds that

$$\lim_{n\to\infty} \boldsymbol{E}[Z_n^r] = \boldsymbol{E}[N^r] \qquad \text{for all} \qquad r \in \mathcal{I},$$

then $\{Z_n\}_{n=1}^{\infty}$ converges to N in distribution.

We should make a remark on this theorem. The case $\mathcal{I} = \{2, 4\}$ is so-called the fourth moment theorem by [3]. The case $\mathcal{I} = \{2, 2k\}$ $(2k \geq 6)$ is shown in [1]. In this paper the authors characterizes the central convergence in terms of some polynomials and show the case (1) generally. This is a great leap in generalization of the fourth moment theorem and part of the case (2) (that is, $\mathcal{I} = \{4, 6\}$, $\{4, 8\}$, $\{4, 10\}$) and (3) are also shown by the same method with (1). The remainder terms (2), (4) and (5) are shown in [2]. Although the proof of them is base on the method developed in [1], we need to combine asymptotic of the hypergeometric function and numerical calculation.

References

[1] Ehsan Azmoodeh, Dominique Malicet, Guillaume Mijoule, and Guillaume Poly. Generalization of the Nualart-Peccati criterion. *Ann. Probab.*, 44(2):924–954, 2016.

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