BV FUNCTIONS AND SETS OF FINITE PERIMETER ON THE CONFIGURATION SPACE

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Abstract

A function *of bounded variation (BV function)* in the Euclidean space $\mathbb{R}^n$ is defined as an integrable function whose weak partial derivative is a (signed) Radon measure with finite total variation. A measurable subset A in $\mathbb{R}^n$ is called a set *of finite perimeter, or Caccioppoli set* if the indicator function χ_A on A is a BV function.

The theory of BV functions provides the study of 'measure-theoretic differentiable functions' and it has been one of the main subjects in geometric measure theory to study properties of singular sets arising in various geometric and analytic contexts, e.g., the theory of minimal surfaces, the structure theory of perimeters, and furthermore, it plays a significant role for stochastic analysis with singular boundaries.

In this talk, we explore the theory of BV functions on the configuration space $\Upsilon(\mathbb{R}^n)$ over the Euclidean space $\mathbb{R}^n$ equipped with the Poisson measure π. As the space $\Upsilon(\mathbb{R}^n)$ is not locally compact and the Poisson measure π does not support the volume doubling property with any reasonable choice of distance functions, there is no chance to apply the existing general theory for BV functions on doubling spaces. In particular, the concept of finite-codimensional measures needs to be rigorously understood in $\Upsilon(\mathbb{R}^n)$. Furthermore, due to the lack of the local compactness, the Riesz–Markov–Kakutani representation theorem is not available, which affects the construction of perimeter measures supporting the Gauß–Green formula.

We start by introducing the concept of the m-codimensional Poisson measure ρ^m on $\Upsilon(\mathbb{R}^n)$, which is formally written as '$(\infty - m)$-dimenisonal Poisson measure'. We then construct ρ^m by using the finite-particle approximation. Based on the measure ρ^m, we develop the theory of BV functions:

- we prove $\mathrm{Cap}_{\alpha,p}(A) = 0 \implies \rho^m(A) = 0$ provided $\alpha p > m$. This is an extension of the well-known relation between (α, p)-Bessel capacities and finite-codimensional Hausdorff measures on $\mathbb{R}^n$ to the case $\Upsilon(\mathbb{R}^n)$;
- we define BV functions in terms of *the variational approach*, *the relaxation approach*, and *the semigroup approach*, and prove the equivalence of them.
- we construct the *total variation measures* and the *perimeter measures* Per_A on a Caccioppoli set A, and prove the *co-area formula*;
- we introduce a measure-theoretic boundary $\partial^* A$ (*reduced boundary*) for a Caccioppoli set A, and prove the *De Giorigi's theorem*:
$$\mathsf{Per}_A = \rho^1|_{\partial^* A};$$
- we prove the *Gauß–Green formula.*

If time allows, we also discuss applications of BV theory to stochastic analysis of infinite particle systems.

This talk is based on the joint work with Elia Brué (Institute for Advanced Study, Princeton).