

Applications of non-local Dirichlet forms defined on infinite dimensional spaces.

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Abstract

By [AKYY 2019] (with detailed proofs), [AK 2019] (with a concise explanation), [Sympo.2019] and [Sympo.2018], general theorems on the closability and quasi-regularity of non-local Markovian symmetric forms on probability spaces $(S, \mathcal{B}(S), \mu)$, with S Fréchet spaces such that $S \subset \mathbb{R}^{\mathbb{N}}$, $\mathcal{B}(S)$ is the Borel σ -field of S , and μ is a Borel probability measure on S , have been introduced. There, a family of non-local Markovian symmetric forms $\mathcal{E}_{(\alpha)}$, $0 < \alpha < 2$, acting in each given $L^2(S; \mu)$ was defined, the index α characterizing the order of the non-locality. Then, it has been shown that all the forms $\mathcal{E}_{(\alpha)}$ defined on $\bigcup_{n \in \mathbb{N}} C_0^\infty(\mathbb{R}^n)$ are closable in $L^2(S; \mu)$. Moreover, sufficient conditions under which the closure of the closable forms, that are Dirichlet forms, become strictly quasi-regular, has been given. Also, an existence theorem for Hunt processes properly associated to the Dirichlet forms has been introduced. In addition, the application of the above abstract theorems to the problem of stochastic quantizations of Euclidean Φ_d^4 fields, for $d = 2, 3$, by means of these Hunt processes has been indicated.

In the present talk, we shall introduce several applications of the abstract theorem to the stochastic quantizations, in the non-local sense, of various random fields:

1. Non-local type stochastic quantization of Euclidean $P(\Phi)_2$ fields.

We consider a *non-local* type stochastic quantization of the *finite* volume Euclidean $P(\Phi)_2$ field.

2. Non-local type stochastic quantization of Euclidean quantum field with exponential potential.

We consider a *non-local* type stochastic quantization of the 2-dimensional Euclidean quantum field with *finite* volume exponential potential.

3. Non-local type stochastic quantization of Euclidean quantum field with trigonometric potentials.

We consider a *non-local* type stochastic quantization of the 2-dimensional Euclidean quantum field with *finite* volume trigonometric potentials.

4. Non-local type stochastic quantization of a field of classical infinite particle system.

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