2019年度　確率解析とその周辺

(Stochastic Analysis and Related Topics 2019)

予稿集

(Abstracts)

2019年11月18日(月) ～ 11月20日(水)

(Nov. 18 (Mon.) – Nov. 20 (Wed.), 2019)

東北大学大学院理学研究科合同C棟2階 青葉サイエンスホール

(Aoba Science Hall, Science Complex C, Aobayama Campus, Graduate School of Science, Tohoku University)

研究集会「確率解析とその周辺」

科学研究費補助金基盤研究 (B) 課題番号 16H03938「無限次元解析の諸問題と確率解析の研究」(研究代表者：会田 茂樹) の援助を受けて、標記の研究集会を以下の要領で開催致しますのでご案内申し上げます。

日時 2019 年 11 月 18 日 (月) 13:20 – 11 月 20 日 (水) 15:40

場所 東北大学大学院理学研究科合同 C 棟 2 階 青葉サイエンスホール
〒 980-8578 宮城県仙台市青葉区荒巻字青葉 6-3

ホームページ https://www.math.kyoto-u.ac.jp/probability/sympo/sa19/

なお 3 日間とも午前中は、松鶴数学講究として Bielefeld 大学の Michael Röckner 氏による講演が以下の通りに行われます。

タイトル Monotonicity methods for SPDE, including the time fractional case.

日時 18 日: 10:30 – 11:30、19 日及び 20 日: 10:00 – 11:30

場所 東北大学数理科学記念館（川井ホール）
〒 980-8578 宮城県仙台市青葉区荒巻字青葉 6-3

ホームページ http://www.math.tohoku.ac.jp/shokaku/index.html

—プログラム—

11 月 18 日 (月)

13:20 – 14:00　松浦浩平 (Kouhei Matsuura)　(Kyoto University)

The strong Feller property of reflected Brownian motions on a class of planar domains.

14:10 – 14:50　土田兼治 (Kaneharu Tsuchida)　(National Defense Academy)

Green-tight measures of Kato class and compact embedding theorem for symmetric Markov processes.

15:10 – 15:50　和田正樹 (Masaki Wada)　(Fukushima University)

Asymptotic behavior of the spectral functions for Schrödinger forms.

16:00 – 16:40　福泉麗佳 (Reika Fukuizumi)　(Tohoku University)

Temperature effects in the model of superfluidity.

16:50 – 17:30　松田響 (Toyomu Matsuda)　(Kyushu University)

Global well-posedness of stochastic complex Ginzburg-Landau equation on the 2D torus.

11 月 19 日 (火)

13:20 – 14:00　星野壮登 (Masato Hoshino)　(Kyushu University)

Paracontrolled calculus and regularity structures.

14:10 – 14:50　謝賓 (Bin Xie)　(Shinshu University)

Asymptotic behavior of reflected SPDEs with singular potentials driven by additive noises.

15:10 – 15:50　河備浩司 (Hiroshi Kawabi)　(Keio University)

Uniqueness of Dirichlet forms related to stochastic quantization of $\exp(\Phi)_2$-measures in finite volume.

16:00 – 16:40　楠岡誠一郎 (Seiichiro Kusuoka)　(Kyoto University)

Stochastic quantization associated with the $\exp(\Phi)_2$-quantum field model driven by space-time white noise on the torus.

16:50 – 17:30　吉田稔 (Minoru W. Yoshida)　(Kanagawa University)

Applications of non-local Dirichlet forms defined on infinite dimensional spaces.

11 月 20 日 (水)

13:20 – 14:00　田口大 (Dai Taguchi)　(Okayama University)

Implicit Euler–Maruyama scheme for radial Dunkl processes.

14:10 – 14:50　結城郷 (Gô Yûki)　(Ritsumeikan University)

Parametrix method for multi-skewed Brownian motion.

15:00 – 15:40　稲浜譲 (Yuzuru Inahama)　(Kyushu University)

Stochastic flows and rough differential equations on foliated spaces.

世話人　会田 茂樹 (東京大学大学院数理科学研究科)
河備 浩司 (慶應義塾大学経済学部)
稲浜 譲（九州大学大学院数理学研究院)
重川 一郎 (京都産業大学理学部)
楠岡 誠一郎（京都大学大学院理学研究科)
永沼 伸顕（大阪大学大学院基礎工学研究科)
針谷 祐 (東北大学大学院理学研究科)

The strong Feller property of reflected Brownian motions on a class of planar domains

Kouhei Matsuura (Kyoto University)

1 Introduction

Gyrya and Saloff-Coste [3] gave two-sided Gaussian heat kernel estimates of the Neumann heat kernels on inner uniform domains. In other words, they showed that the associated Dirichlet spaces satisfy the Poincaré inequality and the volume doubling property. As a corollary, it follows that the Neumann heat kernels are Hölder continuous. Inner uniform domains are generalized notion of uniform domains. For example, the Koch snowflake domain is a typical example of uniform domains. Thus, the boundaries of uniform domains can be fractal sets.

In this talk, we prove the semigroup strong Feller property of Neumann semigroups on a class of planar domains. Domains in the class are not necessarily inner uniform domains. Our proof is mainly based on the conformal invariance of planar reflected Brownian motions and a coupling argument. We also give quantitative lower bounds for Hölder exponents of the Neumann heat kernels on quasidisks.

2 Notation and main results

For a subset $A \subset \mathbb{C}$, we denote by $\overline{A}$ the topological closure in $\mathbb{C}$. We denote by $\mathbb{D}$ the unit disk in $\mathbb{C}$. Let $D \subset \mathbb{C}$ be a Jordan domain. Then, there exists a conformal map $\phi : \mathbb{D} \to D$, which is extended to a homeomorphism from $\overline{\mathbb{D}}$ to $\overline{D}$ by the Carathéodory's theorem. Let $X = (\{X_t\}_{t\in[0,\infty)}, \{P_x^X\}_{x\in\overline{\mathbb{D}}})$ be the reflected Brownian motion on $\overline{\mathbb{D}}$. We define a Hunt process $Y = (\{Y_t\}_{t\in[0,\infty)}, \{P_y^Y\}_{y\in\overline{D}})$ on $\overline{D}$ by

$$Y_t = \phi(X_{A_t^{-1}}),\ P_y^Y = P_{\phi^{-1}(y)}^X, \quad t \in [0,\infty),\ y \in \overline{D}.$$

Here, $\{A_t\}_{t\in[0,\infty]}$ is a positive continuous additive functional of X defined by $A_t = \int_0^t |\phi'(X_s)|^2 \mathbf{1}_D(X_s)\,ds$. Note that A_t strictly increases to ∞ as $t \to \infty$. It is easy to show that the resolvent $\{R_\alpha^Y\}_{\alpha\in(0,\infty)}$ of Y is absolutely continuous with respect to the Lebesgue measure m on $\mathbb{C}$:

$$R_\alpha^Y f(y) = \int_{\overline{D}} r_\alpha^Y(y,z) f(z)\,dm(z), \quad y \in \overline{D},\ \alpha \in (0,\infty),\ f \in \mathcal{B}_{\mathrm{b}}(\overline{D}).$$

Here, $\mathcal{B}_{\mathrm{b}}(\overline{D})$ stands for the space of bounded measurable functions on $\overline{D}$. By [2, Examples 5.3.(2°)], the Dirichlet form $(\mathcal{E},\mathcal{F})$ of Y is regular on $L^2(\overline{D},m)$, which is identified with

$$\mathcal{F} = \{f \in L^2(D,m) \mid |\nabla f| \in L^2(D,m)\}, \quad \mathcal{E}(f,g) = \frac{1}{2}\int_D \langle \nabla f, \nabla g\rangle\,dm, \quad f,g \in \mathcal{F},$$

where ∇f denotes the distributional gradient of f and $\langle\cdot,\cdot\rangle$ denotes the standard inner product on $\mathbb{C}$.

Our main theorem is as follows.

Theorem 1. Suppose that the conformal map $\phi : \mathbb{D} \to D$ is κ-Hölder continuous. Then, for any $\alpha \in (0,\infty)$ and $\varepsilon \in (0,\kappa)$, there exists a constant $C \in (0,\infty)$ such that

$$|R_\alpha^Y f(\phi(x)) - R_\alpha^Y f(\phi(y))| \le C\|f\|_{L^\infty(\overline{D},m)}|x-y|^{(\kappa-\varepsilon)\wedge(1/2)}$$

for any $x,y \in \overline{\mathbb{D}}$, and $f \in \mathcal{B}_{\mathrm{b}}(\overline{D})$. In particular, for any $\alpha \in (0,\infty)$ and $f \in \mathcal{B}_{\mathrm{b}}(\overline{D})$, $R_\alpha^Y f$ is a bounded continuous function on $\overline{D}$.

The image of the unit circle under a quasiconformal mapping on the plane is called a quasicircle. The interior of a quasicircle is called a quasidisk. It is known that quasidisks are uniform domains. Therefore, if D is a quasidisk, we can apply [3, Theorem 3.10] to show that the semigroup $\{P_t^Y\}_{t>0}$ of Y possesses a (unique) continuous kernel $p_t^Y(x,y) : (0,\infty) \times \overline{D} \times \overline{D} \to (0,\infty)$. If D is a quasidisk, it is known that the conformal map $\phi : \mathbb{D} \to D$ is bi-Hölder continuous. Then, Theorem 1 implies that the resolvent $\{R_\alpha^Y\}_{\alpha \in (0,\infty)}$ is also Hölder continuous. Combining these facts with [1, Remark 3.6], we reach the following corollary.

Corollary 1. Suppose that D is a quasidisk. Then, for each $(t,y) \in (0,\infty) \times \overline{D}$, the map $\overline{D} \ni x \mapsto p_t^Y(x,y) \in (0,\infty)$ is Hölder continuous. Furthermore, the Hölder exponent is bounded below by $\lambda\{(\kappa - \varepsilon) \wedge (1/2)\}$, where κ and λ denote the Hölder exponents of ϕ and ϕ^{-1}, respectively, and ε is an arbitrary positive number between 0 and κ.

For a Jordan curve J in $\mathbb{C}$, we define $k(J)$ as

$$k(J) = \inf \frac{|z_1 - z_3||z_2 - z_4|}{|z_1 - z_2||z_3 - z_4| + |z_1 - z_4||z_2 - z_3|} \in [0,1],$$

where the infimum is extended over the set of ordered quadruples z_1, z_2, z_3, z_4 of finite points on J with the property that z_1 and z_3 separate z_2 and z_4 on J.

A Jordan curve J in $\mathbb{C}$ is quasicircle if and only if $k(J) > 0$. In [4, Theorem 1, 2], Näkki and Palka gave estimates for Hölder exponents of conformal maps in terms of $k(J)$. Employing the result, we have

$$\kappa \geq \frac{2 \arcsin^2 k(\partial D)}{\pi(\pi - \arcsin k(\partial D))}, \quad \lambda \geq \frac{\pi}{2(\pi - \arcsin k(\partial D))}.$$

in the situation of Corollary 1.

If the semigroup of Y is ultracontractive, the method of eigenfunction expansion with the resolvent strong Feller property (Theorem 1) immediately imply that the heat kernel of Y has a continuous version on $(0,\infty) \times \overline{D} \times \overline{D}$. However, there are non-inner uniform Jordan domains which satisfy the condition in Theorem 1. In this case, we do not know whether the semigroup of Y is ultracontractive. In the situation of Theorem 1, it is non-trivial even if the semigroup of Y is strong Feller. Then, we establish a Faber–Krahn type inequality for part processes of Y and obtain the following theorem.

Theorem 2. Suppose that the conformal map $\phi : \mathbb{D} \to D$ is Hölder continuous. Then, for any $t > 0$ and $f \in \mathcal{B}_{\mathrm{b}}(\overline{D})$, $P_t^Y f$ is a bounded continuous function on $\overline{D}$.

References

[1] R. F. Bass, M. Kassmann, and T. Kumagai, *Symmetric jump processes: localization, heat kernels and convergence*, Ann. Inst. Henri Poincaré Probab. Stat. **46** (2010), 59–71.

[2] Z.-Q. Chen and M. Fukushima, *Symmetric Markov processes, time change, and boundary theory*, London Mathematical Society Monographs Series, vol. 35, Princeton University Press, Princeton, NJ, 2012.

[3] P. Gyrya and L. Saloff-Coste, *Neumann and Dirichlet heat kernels in inner uniform domains*, Astérisque (2011), viii+144.

[4] R. Näkki and B. Palka, *Quasiconformal circles and Lipschitz classes*, Comment. Math. Helv. **55** (1980), 485–498.

Green-tight measures of Kato class and compact embedding theorem for symmetric Markov processes
(joint work with Kazuhiro Kuwae)

Kaneharu Tsuchida (National Defense Academy)

1 Introduction

Let E be a locally compact separable metric space and m a Radon measure on E with full support. Let $\mathbf{X}$ be a m-symmetric Hunt process and $(\mathcal{E}, \mathcal{F})$ its regular Dirichlet form on $L^2(E;m)$. Takeda proved in [3] that the semigroup of $\mathbf{X}$ is compact on $L^2(E;m)$ if $\mathbf{X}$ satisfies that irreducibility **(I)**, resolvent strong Feller property **(RSF)**, and Green-tightness **(T)**. As an application, the compactness of embedding from $(\mathcal{F}, \mathcal{E})$ to $L^2(E;m)$ can be proved. This compact embedding theorem plays important role in proving the large deviation principle for additive functionals generated by $\mathbf{X}$. In this talk, we extend this result to the Markov process which satisfies absolute continuity **(AC)** and $m \in S^1_{CK_\infty}(\mathbf{X})$, where $S^1_{CK_\infty}(\mathbf{X})$ is a class of Green-tight measures introduced by Chen ([1]). Finally, we present some examples that are **(AC)** but not **(RSF)**.

2 Setting

Let $\mathbf{X} = (\mathbb{P}_x, X_t, \zeta)$ be a m-symmetric special standard process on E, where ζ is the life time of $\mathbf{X}$. Let $(\mathcal{E}, \mathcal{F})$ be the Dirichlet form associated with $\mathbf{X}$, which is known to be quasi-regular. $\mathbf{X}^{(1)}$ denotes the 1-subprocess of $\mathbf{X}$ defined by $\mathbf{X}^{(1)} = (\mathbb{P}_x^{(1)}, X_t)$ with $\mathbb{P}_x^{(1)}(X_t \in A) = e^{-t}\mathbb{P}_x(X_t \in A)$ for all $t > 0$ and $A \in \mathcal{B}(E)$. Let $(P_t)_{t\geq 0}$ be the transition semigroup of $\mathbf{X}$. The transition kernel of $\mathbf{X}$ is denoted by $P_t(x, dy),\ t > 0$, that is, for any $f \in \mathcal{B}_b(E)$,

$$P_t f(x) := \mathbb{E}_x[f(X_t) : t < \zeta] = \int_E f(y) P_t(x, dy), \quad x \in E,\ t > 0.$$

We assume that $\mathbf{X}$ has the absolute continuity condition, that is, for any Borel set B, $m(B) = 0$ implies $P_t(x, B) = \mathbb{P}_x(X_t \in B) = 0$ for all $t > 0$ and $x \in E$. Let $(R_\alpha)_{\alpha > 0}$ be the resolvent of $\mathbf{X}$, that is, for any $f \in \mathcal{B}_b(E)$,

$$R_\alpha f(x) = \mathbb{E}_x\left[\int_0^\infty e^{-\alpha t} f(X_t) dt\right] = \int_0^\infty e^{-\alpha t} P_t f(x) dt.$$

Here $\mathbf{X}$ is said to possess the *resolvent strong Feller property* **(RSF)** (resp. *strong Feller property* **(SF)**) if $R_\alpha(\mathcal{B}_b(E)) \subset C_b(E)$ for any $\alpha > 0$ (resp. $P_t(\mathcal{B}_b(E)) \subset C_b(E)$ for any $t > 0$). It is known that the implication **(SF)** $\Longrightarrow$ **(RSF)** $\Longrightarrow$ **(AC)** holds. Let $S_1(\mathbf{X})$ be the family of positive smooth measures in the strict sense under **(AC)**. For $\nu \in S_1(\mathbf{X})$, we set

$$R_\alpha \nu(x) := \mathbb{E}_x\left[\int_0^\infty e^{-\alpha t} dA_t^\nu\right],\ x \in E,$$

where A_t^ν is the positive continuous additive functional associated to $\nu \in S_1(\mathbf{X})$.

Definition 2.1. A measure $\nu \in S_1(\mathbf{X})$ is said to be in the *Kato class* if $\lim_{\alpha\to\infty} \sup_{x\in E} R_\alpha \nu(x) = 0$. A measure $\nu \in S_1(\mathbf{X})$ is said to be in the *local Kato class* if for any compact subset K of E, $1_K \nu$ is of Kato class.

We denote by $S^1_K(\mathbf{X})$ (resp. $S^1_{LK}(\mathbf{X})$) the family of measures of Kato class (resp. local Kato class).

Definition 2.2 ([1])**.** Let $\nu \in S_1(\mathbf{X})$ and take an $\alpha \geq 0$. When $\alpha = 0$, we always assume the transience of $\mathbf{X}$. ν is said to be an *α-order Green-tight smooth measure of Kato class with respect to* $\mathbf{X}$ if for any

$\varepsilon > 0$ there exists a Borel subset $K = K(\varepsilon)$ of E with $\nu(K) < \infty$ and a constant $\delta > 0$ such that for all ν-measurable set $B \subset K$ with $\nu(B) < \delta$,

$$\sup_{x\in E} R_\alpha(1_{B\cup K^c}\nu) < \varepsilon.$$

In view of the resolvent equation, for a positive constant α, the α-order Green-tightness of Kato class is independent of the choice of $\alpha > 0$. Let denote by $S^1_{CK_\infty}(\mathbf{X})$ (resp. $S^1_{CK^+_\infty}(\mathbf{X})$) the family of 0-order (resp. positive order) Green-tight smooth measure of Kato class. Clearly, $S^1_{CK^+_\infty}(\mathbf{X}) = S^1_{CK_\infty}(\mathbf{X}^{(1)})$. It can be proved as in [1] that $S^1_{CK^+_\infty}(\mathbf{X}) \subset S^1_K(\mathbf{X})$ and $S^1_{CK_\infty}(\mathbf{X}) \subset S^1_K(\mathbf{X})$. It is easy to see that $S^1_{CK_\infty}(\mathbf{X}) \subset S^1_{CK_\infty}(\mathbf{X}^{(1)})$ if $\mathbf{X}$ is transient.

3 Results

Theorem 3.1 ([2, Theorem 1.5]). *Suppose that* $\mathbf{X}$ *satisfies* **(AC)** *and* $m \in S^1_{CK_\infty}(\mathbf{X}^{(1)})$. *Then the embedding* $\mathcal{F} \to L^2(E;m)$ *is compact.*

Theorem 3.2 ([2, Theorem 1.6]). *Suppose that* $\mathbf{X}$ *satisfies* **(AC)** *and* $m \in S^1_{CK_\infty}(\mathbf{X}^{(1)})$. *Then the* L^2*-semigroup* P_t *is a compact operator on* $L^2(E;m)$ *and its every eigenfunction has a finely continuous Borel measurable bounded* m*-version. Moreover, if* $\mathbf{X}$ *satisfies* **(RSF)**, *then every eigenfunction has a bounded continuous* m*-version.*

Let $(\mathcal{F}_e, \mathcal{E})$ be the extended Dirichlet space of $(\mathcal{E}, \mathcal{F})$. For $\nu \in S^1_{CK_\infty}(\mathbf{X})$, the Stollmann-Voigt inequality tells us

$$\int_E f(x)^2\nu(dx) \leq \|R_0\nu\|_\infty \mathcal{E}(f,f), \quad f \in \mathcal{F}_e$$

if $\mathbf{X}$ is transient. This means that $(\mathcal{F}_e, \mathcal{E})$ is continuously embedded in $L^2(E;\nu)$.

Corollary 3.3 ([2, Corollary 1.7]). *Suppose that* $\mathbf{X}$ *is transient and it satisfies* **(AC)**. *Let* $\nu \in S^1_{CK_\infty}(\mathbf{X})$. *Then* $(\mathcal{F}_e, \mathcal{E})$ *is compactly embedded in* $L^2(E;\nu)$.

Let λ_2 be the bottom of the spectrum:

$$\lambda_2 := \inf\left\{\mathcal{E}(f,f) : f \in \mathcal{F},\ \int_E f^2 dm = 1\right\}.$$

A function ϕ_0 on E is called a *ground state* of the L^2-generator for $\mathcal{E}$ if $\phi_0 \in \mathcal{F}$, $\|\phi_0\|_2 = 1$ and $\mathcal{E}(\phi_0, \phi_0) = \lambda_2$.

Theorem 3.4 ([2, Theorem 1.8]). *Suppose that* $\mathbf{X}$ *satisfies* **(AC)**, **(I)** *and* $m \in S^1_{CK_\infty}(\mathbf{X}^{(1)})$. *Then there exists a bounded ground state* ϕ_0 *uniquely up to sign. Moreover,* ϕ_0 *can be taken to be strictly positive on* E.

Finally, we state the following general theorem to construct examples which do not possess **(RSF)**, but satisfy **(AC)**.

Theorem 3.5 ([2, Theorem 4.1, 4.2]). *Suppose that* $\mathbf{X}$ *is transient and possesses* **(RSF)**. *Take* $\nu \in S_1(\mathbf{X})$ *with* $\|R_0\nu\|_\infty < \infty$ *and assume* $\nu \notin S^1_{LK}(\mathbf{X})$. *If* ν *has the full quasi-support, then the time-changed process* $(\check{\mathbf{X}}, \nu)$ *does not possess* **(RSF)**, *but satisfies* **(AC)**. *Under same assumptions on* ν, *there exists an* $\alpha > 0$ *such that the killed process* $\mathbf{X}^{-\alpha\nu}$ *does not possess* **(RSF)**, *but satisfies* **(AC)**.

We will give concrete examples during the talk.

References

[1] Z.-Q. Chen: Gaugeability and conditional gaugeability, *Trans. Amer. Math. Soc.* **354** (2002), 4639–4679.

[2] K. Kuwae and K. Tsuchida: Green-tight measures of Kato class and compact embedding theorem for symmetric Markov processes, preprint, 2019.

[3] M. Takeda: Compactness of symmetric Markov semi-groups and boundedness of eigenfunctions, *Trans. Amer. Math. Soc.* **372** (2019), 3905–3920.

Asymptotic behavior of the spectral functions for Schrödinger forms

Masaki Wada (Fukushima University)

October 31, 2019

1 Setting and the main result

Let $\{X_t\}$ be the rotationally invariant α-stable process on $\mathbb{R}^d$ with $0 < \alpha < 2$ and denote by $(\mathcal{E}, \mathcal{F})$ the corresponding Dirichlet form on $L^2(\mathbb{R}^d)$. We assume $\alpha < d$, transience of $\{X_t\}$ and denote the Green kernel by $G(x, y)$. Let μ and ν be positive Radon smooth measures satisfying three properties, i.e. Kato class, Green tightness and of finite 0-order energy integral. Define the Schrödinger form by

$$\mathcal{E}^\lambda(u, v) = \mathcal{E}(u, v) - \int_{\mathbb{R}^d} u(x)v(x)\mu(dx) - \lambda \int_{\mathbb{R}^d} u(x)v(x)\nu(dx) \qquad (\lambda \geq 0)$$

For simplicity, we also assume μ is critical, that is,

$$\inf\left\{\mathcal{E}(u, u) \;\middle|\; u \in \mathcal{F}_e, \quad \int_{\mathbb{R}^d} u^2(x)\mu(dx) = 1\right\} = 1.$$

Here $\mathcal{F}_e$ is the extended Dirichlet space. Define the spectral function by

$$C(\lambda) = -\left\{\mathcal{E}^\lambda(u, u) \;\middle|\; \int_{\mathbb{R}^d} u^2(x)dx = 1\right\}.$$

There are several preceding results for the differentiability of the spectral functions. Takeda and Tsuchida [2] treated this problem in the framework of $\mu = \nu$. Nishimori [1] treated the differentiability of $C(\lambda)$. Both of them showed that the differentiability of the spectral function is equivalent to $d/\alpha \leq 2$. In this talk, we treat the precise asymptotic behavior of the spectral function and our main result is as follows:

Theorem 1. *(W. 2018)*
As $\lambda \downarrow 0$, the spectral function $C(\lambda)$ satisfies the asymptotic behavior as follows:

$$C(\lambda) \sim \left(\frac{\alpha\Gamma(\frac{d}{2})|\sin(\frac{d}{\alpha}\pi)|\langle h_0, h_0\rangle_\nu}{2^{1-d}\pi^{1-\frac{d}{2}}\langle \mu, h_0\rangle^2}\lambda\right)^{\frac{\alpha}{d-\alpha}} \qquad (1 < d/\alpha < 2)$$

$$C(\lambda) \sim \frac{\Gamma(\alpha+1)\langle h_0, h_0\rangle_\nu}{2^{1-d}\pi^{-\frac{d}{2}}\langle \mu, h_0\rangle^2} \cdot \frac{\lambda}{\log \lambda^{-1}} \qquad (d/\alpha = 2)$$

$$C(\lambda) \sim \frac{\langle h_0, h_0\rangle_\nu}{\langle h_0, h_0\rangle_m} \cdot \lambda \quad (d/\alpha > 2)$$

Here $h_0(x)$ is the ground state of $\mathcal{E}^0$ and m stands for the Lebesgue measure of $\mathbb{R}^d$.

Remark 2. For $\mu = \nu = V \cdot m$, this result is the same as in [3]

2 Outline of the proof

(1) Let $G_\beta(x, y)$ be the resolvent kernel of $\{X_t\}_{t\geq 0}$. Define the compact operators by

$$K_\lambda f(x) = \int_{\mathbb{R}^d} G_{C(\lambda)}(x, y) f(y)(\mu + \lambda\nu)(dy) \quad f \in L^2(\mu + \lambda\nu)$$

$$\tilde{K}_\lambda f(x) = \int_{\mathbb{R}^d} G_{C(\lambda)}(x, y) f(y)\mu(dy) \quad f \in L^2(\mu)$$

(2) Denote the principal eigenfunction of these operator by h_λ and $\tilde{h}_\lambda$. The principal eigenvalue of K_λ is 1, while the principal eigenvalue of $\tilde{K}_\lambda$ admits the asymptotic behavior as follows.

$$\lim_{\lambda\to 0} \frac{1 - \gamma_{C(\lambda)}}{k(C(\lambda))} = \kappa(d, \alpha, \mu) \qquad k(\beta) = \begin{cases} \beta^{d/\alpha - 1} & (1 < d/\alpha < 2) \\ \beta \log \beta^{-1} & (d/\alpha = 2) \\ \beta & (d/\alpha > 2) \end{cases}$$

Here $\kappa(d, \alpha, \mu)$ is a unique positive constant.

(3) Considering the inner product of h_λ and $\tilde{h}_\lambda$, we have

$$(1 - \gamma_{C(\lambda)})\langle h_\lambda, \tilde{h}_\lambda\rangle_\mu = \lambda\langle h_\lambda, \tilde{h}_\lambda\rangle_\nu.$$

Both h_λ and $\tilde{h}_\lambda$ converges to the ground state h_0 in $L^2(\mu)$ and $L^2(\nu)$. Thus we obtain the desired result.

References

[1] Nishimori, Y.: Large deviations for symmetric stable processes with Feynman-Kac functionals and its application to pinned polymers, Tohoku Math. Journal 65, 467–494, (2013).

[2] Takeda, M. and Tsuchida, K.: Differentiability of spectral functions for symmetric α-stable processes, Trans. Amer. Math. 359, 4031–4054, (2007).

[3] Wada, M.: Asymptotic expansion of resolvent kernels and behavior of spectral functions for symmetric stable processes, J. Math. Soc. Jpn. 69, 673–692, (2017).

Temperature effects in the model of superfluidity

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Abstract

The stochastic Gross-Pitaevskii equation is used as a model of Bose-Einstein condensation (BEC) at positive temperature. The equation is a complex Ginzburg- Landau equation with a trapping potential and an additive space-time white noise. A positive temperature effect, for example, the spontaneous vortex formation by a sudden quench in BEC (seen as a phase transition) is of great interest in Physics, and the Gibbs equilibrium is the key ingredient in the analysis from the point of view in statistical physics. In this talk we will first give some recent results on the 2D stochastic Gross-Pitaevskii equation, where a Wick *inhomogeneous* renormalization is required to give a sense to the nonlinearity. We will refer to another result on a closely related, but another model described by a stochastic damped nonlinear wave equation too if time permitting. This talk will be based on joint works with Anne de Bouard (Ecole polytechnique), Arnaud Debussche (ENS Rennes), and Masato Hoshino (Kyushu University).

Global well-posedness of stochastic complex Ginzburg-Landau equation on the 2D torus

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In this talk, we study the following stochastic complex Ginzburg-Landau equation

$$\begin{cases} \partial_t u = (i+\mu)\Delta u - \nu |u|^2 u + \lambda u + \xi, & t > 0,\, x \in \mathbb{T}^2, \\ u(0,\cdot) = u_0, \end{cases} \tag{1}$$

where $\mu > 0$, $\nu \in \{z \in \mathbb{C} \,|\, \Re z > 0\}$, $\lambda \in \mathbb{C}$ and $\mathbb{T}^2$ is a two dimensional torus. The random field ξ is the complex space-time white noise, i.e., the centered Gaussian random field with covariance structure

$$\mathbb{E}[\xi(t,x)\xi(s,y)] = 0, \qquad \mathbb{E}[\xi(t,x)\overline{\xi(s,y)}] = \delta(t-s)\delta(x-y).$$

The main objective is to discuss global well-posedness of this equation and Markov properties of the solution.

The difficulty lies in the fact that a solution of the equation (1) has to be a Schwartz distribution due to the low regularity of the noise ξ, which leaves the nonlinear term $|u|^2 u$ ill-defined. To overcome this difficulty, we employ a strategy first introduced in [DD03]. Namely, we decompose a solution $u = Z + Y$, where Z is a solution of a linear SPDE

$$\partial_t Z = (i+\mu)\Delta Z - Z + \xi.$$

Although Z is still a distibution-valued random variable, we can naturally define its products via Wick renormalization.

Then Y formally solves

$$\begin{aligned}\partial_t Y &= (i+\mu)\Delta Y - Y \\ &\quad + (1+\lambda)(Z+Y) - \nu(|Y|^2 Y + 2Z|Y|^2 + \overline{Z}Y^2 + 2|Z|^2 Y + Z^2\overline{Y} + |Z|^2 Z).\end{aligned} \tag{2}$$

In the light of Shauder's estimate, Y_t has positive regularity and therefore the products appearing in the equation (2) are well-defined.

The main theorem of this talk is the following;

Theorem 1. *There exists a unique (mild) solution of* (2) *over any time interval* $[0,T]$.

For the proof of the theorem, we follow the paper [MW17], where the authors show global well-posedness of the dynamic Φ^4_2 equation. In our setting, however, the argument in [MW17] does not imply a priori L^p estimates of solutions for large p. We overcome this obstacle by bootstrap arguments, which leads to;

Theorem 2. *Let* $\beta \in (0,2)$ *and* $\mathscr{C}^\beta := \mathscr{B}^\beta_{\infty,\infty}$ *be a Besov space. Then there exists* $\kappa \in (0,\infty)$ *such that for every solution* Y *of* (2) *over* $[0,T]$,

$$\sup_{t_0 \le t \le T} \|Y_t\|_{\mathscr{C}^\beta} \lesssim_{Z,\beta,T} t_0^{-\kappa},$$

uniformly for the initial value Y_0.

Finally, I discuss my ongoing research on Markov properties of the solution as in [TW18].

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PARACONTROLLED CALCULUS AND REGULARITY STRUCTURES

MASATO HOSHINO

Regularity structure (RS) by Hairer (2014) and *Paracontrolled calculus* (PC) by Gubinelli-Imkeller-Perkowski (2015) both solve many singular SPDEs. These theories are believed to be equivalent, but there are some gaps. For example, the general KPZ equation

$$\partial_t h = \partial_x^2 h + f(h)(\partial_x h)^2 + g(h)\xi$$

is solved by RS, but cannot be solved within PC. Our aim is to show the equivalence between RS and (an extension of) PC and fill such gaps.

Both of RS and PC are extensions of the *rough path theory*. Our main result means that the two different ways of defining the "rough paths" on the same algebraic structure are equivalent. A rough image of our main result is the following.

Theorem 1. [1, 2] *The following equivalences hold.*

Rough path theory	*RS*		*PC*
Rough path	*Model*	$\Leftrightarrow$ [1]	*Pararemainders*
Controlled path	*Modelled distribution*	$\Leftrightarrow$ [2]	*Paracontrolled distribution*

We explain the precise meanings. Recall that the α-Hölder geometric rough paths living in $\mathbb{R}^n$ are α-Hölder continuous paths on the $\lfloor 1/\alpha \rfloor$-step nilpotent Lie group $G^{(\lfloor 1/\alpha \rfloor)}(\mathbb{R}^n)$, which is identified with a space of linear functionals g on $T^{(\lfloor 1/\alpha \rfloor)}(\mathbb{R}^n)$ such that

$$g(x \sqcup\!\sqcup y) = g(x)g(y)$$

for any $x, y \in T^{(\lfloor 1/\alpha \rfloor)}(\mathbb{R}^n)$, where $\sqcup\!\sqcup$ is the shuffle product. The model is defined similarly by replacing $T^{(\lfloor 1/\alpha \rfloor)}(\mathbb{R}^n)$ by a general graded Hopf algebra T^+.

Definition 1. *A* (concrete) regularity structure *is a pair of a graded Hopf algebra* $T^+ = \bigoplus_{\alpha \in A^+} T_\alpha^+$ *with a coproduct* Δ^+ *and a graded linear space* $T = \bigoplus_{\beta \in A} T_\beta$ *with a linear map* $\Delta : T \to T \otimes T^+$ *satisfying the right comodule properties on* T^+ *such that*

$$\Delta^+ \tau \in \tau \otimes \mathbf{1} + \mathbf{1} \otimes \tau + \bigoplus_{0<\beta<\alpha} T_\beta^+ \otimes T_{\alpha-\beta}^+,$$

$$\Delta \sigma \in \sigma \otimes \mathbf{1} + \bigoplus_{\gamma<\beta} T_\gamma \otimes T_{\beta-\gamma}^+,$$

for any $\tau \in T_\alpha^+$ *and* $\sigma \in T_\beta$.

Let $\mathcal{B}_\alpha^{(+)}$ be a basis of $T_\alpha^{(+)}$ and let $\mathcal{B}^{(+)} = \bigcup_{\alpha \in A^{(+)}} \mathcal{B}_\alpha^{(+)}$. We assume that the basis $\mathcal{B}^+$ is a monoid generated by $\mathcal{B}_\circ^+$. Moreover, we assume that $\mathcal{B}_\circ^+$ consists of

- monomials $X_1, \dots, X_d$,
- derivatives $\partial^k \tau$ of "pure" elements $\tau \in \mathcal{P}\mathcal{B}_\circ^+$.

Then we have the following equivalence result. We say that each element $\tau \in T_\alpha^{(+)}$ is homogeneous and write

$$|\tau| = \alpha.$$

Theorem 2. [1, 2] *Let $\mathcal{M}_{\rm rap}$ be the space of all* models (g, Π) *on $\mathbb{R}^d$, i.e., all pairs of*

- *a Lipschitz continuous map g from $\mathbb{R}^d$ to the group G of algebra homomorphisms $T^+ \to \mathbb{R}$,*
- *a bounded operator $\Pi : T \to \mathcal{S}'(\mathbb{R}^d)$ such that $\Pi\tau$ has a "$|\tau|$-class Taylor expansion" for any homogeneous $\tau \in T$,*

and such that $\mathsf{g}(\tau)$ and $\Pi\sigma$ rapidly decrease at infinity for any $\tau \in T^+$ and $\sigma \in T$. Then the space $\mathcal{M}_{\rm rap}$ is homeomorphic to the direct product of Banach spaces;

$$\mathcal{M}_{\rm rap} \simeq \prod_{\tau \in \mathcal{B}_\circ,\, |\tau|<0} \mathcal{C}_{\rm rap}^{|\tau|}(\mathbb{R}^d) \times \prod_{\sigma \in \mathcal{PB}_\circ^+} \mathcal{C}_{\rm rap}^{|\sigma|}(\mathbb{R}^d).$$

We return to the rough path theory. The path $Y : [0, T] \to \mathbb{R}$ is said to be an α-Hölder path controlled by a rough path $\mathbf{X}$ if there exists a continuous path $\mathbf{Y} : [0, T] \to T^{(\lfloor 1/\alpha \rfloor - 1)}(\mathbb{R}^n)$ such that $Y_t^\emptyset = Y_t$ and

$$Y_t^{i_1 \dots i_k} = \sum_{i_{k+1}, \dots, i_\ell} Y_s^{i_1 \dots i_k i_{k+1} \dots i_\ell} \mathbf{X}_{st}^{i_1 \dots i_k i_{k+1} \dots i_\ell} + O(|t-s|^{(\lfloor 1/\alpha \rfloor - k)\alpha})$$

for any $i_1, \dots, i_k \in \{1, \dots, n\}$, where $Y^{i_1 \dots i_k}$ and $\mathbf{X}^{i_1 \dots i_k}$ represents the $e_{i_1} \otimes \cdots \otimes e_{i_k}$-components of Y and $\mathbf{X}$, respectively. A modelled distribution is a T-valued function on $\mathbb{R}^d$ with a similar "Taylor-like expansion" at each point $x \in \mathbb{R}^d$. Note that any $\mathsf{g} \in G$ acts on T by

$$\hat{\mathsf{g}}(\tau) := (\mathrm{Id} \otimes \mathsf{g})\Delta\tau, \quad \tau \in T.$$

Denote by $(\cdot)_\tau : T \to \mathbb{R}$ the projection to the τ-component.

Theorem 3. [2] *Let $\gamma \in \mathbb{R}$. Let $\mathcal{D}_{\rm rap}^\gamma$ be the space of all* γ-class modelled distributions*, i.e., all functions $f : \mathbb{R}^d \to T$ such that, for any homogeneous $\tau \in T$,*

$$\big(f(y) - \hat{\mathsf{g}}_{yx} f(x)\big)_\tau = O(|y-x|^{\gamma - |\tau|}),$$

where $\mathsf{g}_{yx} := \mathsf{g}_y \mathsf{g}_x^{-1} \in G$, and $\big(f(x)\big)_\tau$ rapidly decreases as $|x| \to \infty$.

We assume that the basis $\mathcal{B}$ of T has a good structure; monomials and antiderivatives. Then the space $\mathcal{D}_{\rm rap}^\gamma$ is homeomorphic to the direct product of Banach spaces;

$$\mathcal{D}_{\rm rap}^\gamma \simeq \prod_{\tau \in \mathcal{PB}_\circ,\, |\tau|<\gamma} \Big(\prod_{\eta \in \mathcal{PB}_\tau} \mathcal{C}_{\rm rap}^{\gamma - |\eta|}(\mathbb{R}^d) \Big),$$

where $\mathcal{PB}_\circ$ is the set of all "pure" elements, and $\mathcal{PB}_\tau$ is the set of independent antiderivatives of τ. (If the antiderivative is unique for each τ, then $\mathcal{PB}_\tau = \{\tau\}$.)

This talk is based on a joint work with Ismaël Bailleul (Université de Rennes 1).

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Asymptotic behavior of reflected SPDEs with singular potentials driven by additive noises

Bin Xie (Shinshu University)

In this talk, we will mainly review the results on the asymptotic behavior of the following reflected SPDE with a singular potential by making use of the dimension-free Harnack inequality:

$$\frac{\partial u}{\partial t}(t,\theta) = -\frac{1}{2}(-\Delta)^{\gamma}\left(\frac{\delta \mathcal{H}(u)}{\delta u(\theta)}(t,\theta) + \xi(t,\theta)\right) + (-\Delta)^{\gamma/2} B\dot{W}(t,\theta). \qquad (1)$$

Hereafter, $\gamma \in \{0,1\}$, $\theta \in (0,1)$, $\xi(t,\theta)$ denotes a signed random reflecting measure which prevents the solution from leaving a subset $\mathcal{I}$ of $\mathbb{R}$, $\dot{W}(t,\theta)$ denotes a Gaussian noise and $\frac{\delta \mathcal{H}(u)}{\delta u(\theta)}$ denotes the functional derivative of the formal Hamiltonian $\mathcal{H}$, where

$$\mathcal{H}(u) = \int_0^1 \left(\frac{1}{2}|\nabla u(\theta)|^2 + F(u(\theta))\right) d\theta$$

is called the Ginzburg-Landau-Wilson free energy with a self-potential F.

The SPDE (1) is sometimes called the time-dependent Ginzburg-Landau equation. In addition, because the reflecting term $\xi(t,\theta)$ is considered, (1) is also called the random parabolic obstacle problem and is regarded as the infinite-dimensional Skorokhod problem. Formally, we can easily see that the mass of solutions of (1) (i.e. $\int_0^1 u(t,\theta)d\theta$)) is conservative under the time evolution for the case $\gamma = 1$, whereas it is not conservative for the case $\gamma = 0$. Hence, according to Hohenberg and Halperin (1977), we will call (1) with $\gamma = 0$ the Model A with reflection, and respectively (1) with $\gamma = 1$ the Model B with reflection in the following. As for applications, the reflected SPDE (1) has been used to model the fluctuations for $\nabla\phi$ interface models on a hard wall with or without conservation of the area. In this talk, both the Model A and the Model B will be discussed.

For the Model A with reflection, we will study (1) with $\mathcal{I} = [0,\infty)$ and $B = I$ under Dirichlet boundary conditions. Noting that $\frac{\delta \mathcal{H}(u)}{\delta u(\theta)} = -\Delta u(\theta) + F'(u(\theta))$, we know that, under the above assumptions, (1) can be written as

$$\begin{cases} \dfrac{\partial u}{\partial t}(t,\theta) = \dfrac{1}{2}\dfrac{\partial^2 u}{\partial \theta^2}(t,\theta) - F'(u(t,\theta)) + \dot{W}(t,\theta) + \xi(dtd\theta),\ \theta \in (0,1), \\ u(t,0) = u(t,1) = 0, \\ u(0,\theta) = h(\theta) \geq 0,\ \theta \in (0,1), \\ u(t,\theta) \geq 0,\ \theta \in [0,1]\ a.s, \end{cases} \qquad (2)$$

which is called the reflected stochastic reaction-diffusion equation. Under very weak conditions, we study the hypercontractive property of the Markov semigroup associated with (2) driven by the additive space-time white noise. To show it, the coupling of property, the Harnack inequality with power of the Markov semigroup and the Gaussian concentration of the invariant probability measure are investigated respectively. As the same time, we can show the compactness of the Markov semigroup, the exponential convergences of the Markov semigroup to its unique invariant measure in the sense of L^2, total variation norm and entropy are obtained.

In order to explain the Model B with reflection, let us first define the non-linear function f of the logarithmic type by

$$f(u) = \log\left(\frac{1-u}{1+u}\right) + \lambda u,\ u \in (-1,1),$$

which has two singularity points -1 and 1 and let $F'(u) = -f(u)$. Then (1) with $\mathcal{I} = [-1,1]$ is called the stochastic Cahn-Hilliard equation with logarithmic potential and particularly under Neumann boundary conditions, it can be written as the following:

$$\begin{cases} \dfrac{\partial u}{\partial t}(t,\theta) =& -\dfrac{1}{2}\dfrac{\partial^2}{\partial\theta^2}\left(\dfrac{\partial^2 u}{\partial\theta^2}(t,\theta) + f(u(t,\theta)) + \xi(t,\theta)\right) + B\dot{W}(t,\theta), \\ u(t,0) =& u(t,1) = \dfrac{\partial^3 u}{\partial x^3}(t,0) = \dfrac{\partial^3 u}{\partial x^3}(t,1) = 0,\ t \geq 0, \\ u(0,\theta) =& x(\theta),\ \theta \in (0,1), \\ u(t,\theta) \in& [-1,1]\ a.s., \end{cases} \tag{3}$$

where $\xi(t,\theta) = \eta_-(t,\theta) - \eta_+(t,\theta)$ preventing the solution $u(t,\theta)$ from exiting $[-1,1]$. In fact, in applications, the solution of (3) is explained as the rescaled density of atoms or concentration of one of material's components which naturally takes values in $[-1,1]$. Hence, the function f defined above is more important and owing to the effect of noises, reflecting measures $\eta_-(t,\theta)$ and $\eta_+(t,\theta)$ are required. We will study the asymptotic behavior of the solution of (3) driven by both the degenerate colored noise and the non-degenerate white noise. For the case of degenerate colored noise, the asymptotic log-Harnack inequality is established under the so-called essentially elliptic conditions, which implies the asymptotic strong Feller property. For the case of non-degenerate space-time white noise, the Harnack inequality with power is established. This part is the joint work with L. Goudenège.

Uniqueness of Dirichlet forms related to stochastic quantization of $\exp(\Phi)_2$-measures in finite volume

Hiroshi KAWABI (Keio University) *

This talk is based on a (still ongoing) joint work with Sergio Albeverio (Universität Bonn), Stefan Mihalache (KPMG, Frankfurt) and Michael Röckner (Universität Bielefeld). In this talk, we discuss L^p-uniqueness for the diffusion operators defined through Dirichlet forms given by space-time quantum fields with interactions of exponential type, called $\exp(\Phi)_2$-*measures* (Høegh-Krohn's model of quantum fields), in finite volume.

Let $\mathbb{T}^2 = (\mathbb{R}/2\pi\mathbb{Z})^2$ be the two dimensional torus and $H^s(\mathbb{T}^2)$, $s \in \mathbb{R}$ denotes the Sobolev space of order s with periodic boundary condition. We put $H := L^2(\mathbb{T}^2)$. Let μ_0 be the mean-zero Gaussian measure on $E := H^{-\beta}(\mathbb{T}^2)$, $\beta > 0$ with covariance operator $(1-\Delta)^{-1}$. It is called the (massive) *Gaussian free field* (in finite volume). For a charge parameter $\alpha \in (-\sqrt{4\pi}, \sqrt{4\pi})$ and a Gaussian free field z, we formally introduce a random measure $\mathcal{M}_z^{(\alpha)}$ on $\mathbb{T}^2$ by

$$\mathcal{M}_z^{(\alpha)}(dx) := \exp^{\diamond}(\alpha z(x))dx = \exp\Big(\alpha z(x) - \frac{\alpha^2}{2}\mathbb{E}^{\mu_0}\big[z(x)^2\big]\Big)dx, \quad x \in \mathbb{T}^2.$$

This measure is called the the *Liouville measure* in the context of Liouville quantum gravity. We then define the $\exp(\Phi)_2$-measure $\mu = \mu_{\mathsf{exp}}^{(\alpha)}$ by

$$\mu(dz) = Z_\alpha^{-1} \exp(-\mathcal{M}_z^{(\alpha)}(\mathbb{T}^2))\mu_0(dz),$$

where $Z_\alpha > 0$ is the normalizing constant. It is a probability measure on E.

We now fix $\gamma > 0$ such that $\beta + 2\gamma > 2$, and consider a pre-Dirichlet form $(\mathcal{E}, \mathcal{FC}_b^\infty)$ which is given by

$$\mathcal{E}(F,G) = \frac{1}{2}\int_E \Big((1-\Delta)^{-\gamma} D_H F(z), D_H G(z)\Big)_H \mu(dz), \quad F, G \in \mathcal{FC}_b^\infty,$$

where $\mathcal{FC}_b^\infty$ is the set of all smooth cylindrical functions on E and D_H denotes the H-Fréchet derivative. The corresponding pre-Dirichlet operator $(\mathcal{L}_0, \mathcal{FC}_b^\infty)$ is defined by $\mathcal{E}(F,G) = -(\mathcal{L}_0 F, G)_{L^2(\mu)}$, $F, G \in \mathcal{FC}_b^\infty$. It implies that $(\mathcal{E}, \mathcal{FC}_b^\infty)$ is closable on $L^2(\mu)$. We denote the closure of $(\mathcal{E}, \mathcal{FC}_b^\infty)$ by $(\mathcal{E}, \mathcal{D}(\mathcal{E}))$.

Our main theorem in this talk is the following. (If time permits, I will touch a sketch of the proof based on the argument in [LR98].)

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Theorem. *Assume $\beta, \gamma > 0$, $\beta + 2\gamma > 2$ and $|\alpha| < \min\{\sqrt{4\pi\gamma}, \sqrt{4\pi}\}$. Then the pre-Dirichlet operator $(\mathcal{L}_0, \mathcal{F}\mathcal{C}_b^\infty)$ is $L^p(\mu)$-unique for all $1 \le p < \frac{1}{2}\big(1 + \frac{4\pi\gamma}{\alpha^2}\big)$. Namely, there exists exactly one C_0-semigroup on $L^p(\mu)$ such that its generator extends $(\mathcal{L}_0, \mathcal{F}\mathcal{C}_b^\infty)$. In particular, we obtain the Markov uniqueness, that is, the Dirichlet form $(\mathcal{E}, \mathcal{D}(\mathcal{E}))$ is the unique extension of $(\mathcal{E}, \mathcal{F}\mathcal{C}_b^\infty)$ such that $\mathcal{F}\mathcal{C}_b^\infty$ is contained in the domain of the associated generator.*

We remark that L^1-uniqueness for this model and L^p-uniqneness for $P(\Phi)_1$-, $\exp(\Phi)_1$-models in *infinite* volume have been obtained in [Wu00] and [AKR12], respectively.

As an application of this theorem, we have the *unique* existence of *weak* solution to the corresponding modified-stochastic quantization equation:

$$\begin{aligned}\partial_t u(t,x) = &-\frac{1}{2}(1-\Delta)^{1-\gamma}u(t,x) - \frac{\alpha}{2}(1-\Delta)^{-\gamma}\exp\Big(\alpha u(t,x) - \frac{\alpha^2}{2}\infty\Big) \\ &+ (1-\Delta)^{-\frac{\gamma}{2}}\xi(t,x), \qquad t>0,\ x\in\mathbb{T}^2,\end{aligned}$$

where $\xi = (\xi(t,x))_{t\ge 0, x\in\mathbb{T}^2}$ is a space-time white noise on $[0,\infty)\times\mathbb{T}^2$. Furthemore, by following the argument in [HKK19], we may construct a unique *strong* solution to this singular SPDE. However, it does not imply the $L^p(\mu)$-uniqueness of the Dirichlet operator. This is obvious, since a priori the latter might have extensions which generate non-Markovian semigroups which thus have no probabilistic interpretation as transition probabilities of a process. Therefore, neither of $L^p(\mu)$-uniqueness of the Dirichlet operator and strong uniqueness of the corresponding SPDE implies the other.

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Stochastic quantization associated with the $\exp(\Phi)_2$-quantum field model driven by space-time white noise on the torus

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This is a joint work with Masato Hoshino and Hiroshi Kawabi. Let Λ be the 2-dimensional torus $(\mathbb{R}/2\pi\mathbb{Z})^2$ and μ_0 be Nelson's free field measure on Λ with mass 1. We consider the $\exp(\Phi)_2$-quantum field measure:

$$\mu_{\exp}^{(\alpha)}(d\phi) = \frac{1}{Z^{(\alpha)}} \exp\left(- \int_\Lambda \exp^\diamond(\alpha\phi)(x)dx \right) \mu_0(d\phi)$$

where $Z^{(\alpha)}$ is the normalizing constant and $\exp^\diamond(\alpha\phi)$ is the Wick exponential defined by

$$\exp^\diamond(\alpha\phi) = :\exp(\alpha\phi): = \sum_{n=0}^\infty \frac{\alpha^n}{n!} :\phi^n: .$$

We also have a formal equation

$$\exp^\diamond(\alpha\phi) = \exp\left(\alpha\phi - \frac{\alpha^2}{2} E^{\mu_0}[\phi(\cdot)^2] \right).$$

Here, note that $E^{\mu_0}[\phi(\cdot)^2] = \infty$. In this talk, we consider the stochastic quantization of the $\exp(\Phi)_2$-measure, which is a time-evolution having the $\exp(\Phi)_2$-measure as an invariant measure. The associated SPDE is given by

$$\partial_t \Phi_t(x) = \frac{1}{2}(\triangle - 1)\Phi_t(x) - \frac{\alpha}{2}\exp^\diamond(\alpha\Phi_t(x)) + \dot{W}_t(x), \quad x \in \Lambda \tag{1}$$

where $\dot{W}_t(x)$ is a white noise with parameter (t, x). This SPDE is obtained by log-derivative of $\mu_{\exp}^{(\alpha)}$. Let $X = X(\phi)$ be the infinite-dimensional Ornstein-Uhlenbeck process, which is the solution to

$$\begin{cases} \partial_t X_t &= \frac{1}{2}(\triangle - 1)X_t + \dot{W}_t \\ X_0 &= \phi. \end{cases}$$

If Φ is a solution to (1) in some sense, $Y := \Phi - X$ satisfies

$$\partial_t Y_t = \frac{1}{2}(\triangle - 1)Y_t - \frac{\alpha}{2}\exp^\diamond(\alpha X_t)\exp(\alpha Y_t). \tag{2}$$

We call (2) the shifted equation of (1). Via approximations for $|\alpha| < \sqrt{4\pi}$ we are able to define the process of Wick exponentials of X_t by

$$\mathcal{X}_t(\phi) = \exp^\diamond(\alpha X_t(\phi))$$

and

$$\int_{\mathcal{D}'(\Lambda)} \left(\int_0^T \|\mathcal{X}_t(\phi)\|_{H^{-\beta}(\Lambda)}^2 \, dt \right) \mu_0(d\phi) < \infty.$$

for $\beta \in (\alpha^2/(4\pi), 1)$. There are some difficulties to apply general theories such as the regularity structure and the paracontrolled calculus to (1). When we apply a general theory, we usually assume that inputs (driving processes) are $W^{s,\infty}$-valued processes. However, $\exp^\diamond(\alpha X_t)$ is a $W^{-\alpha^2/(4\pi)-\varepsilon,2}$-valued process, and to improve the integrability 2 we need to loose the regularity. The regularity is very serious for singular SPDEs. So, we need to solve (1) in the spaces with integrability 2 by using a model dependent argument. Another difficulty is that exponential functions do not have polynomial growth, and the derivatives are unbounded. These are the reasons that we study the $\exp(\Phi)_2$-model by using the method by singuar SPDEs. We remark that $\mathcal{X}_t(\phi)$ is a nonnegative distribution almost every ϕ, and that the nonnegativity plays an important role in the proofs of main theorems.

Theorem 1. *Assume that* $|\alpha| < \sqrt{4\pi}$ *and let* $\beta \in (\alpha^2/(4\pi), 1)$, $\mathcal{X} \in L^2([0,T]; H_+^{-\beta})$ *and* $v \in H^{2-\beta}$. *Then,*

$$\begin{cases} \partial_t \mathcal{Y}_t &= \frac{1}{2}(\triangle - 1)\mathcal{Y}_t - \frac{\alpha}{2} e^{\alpha \mathcal{Y}_t} \mathcal{X}_t \\ \mathcal{Y}_0 &= v, \end{cases}$$

has a unique mild solution in $L^2([0,T]; H^{1+\delta}) \cap C([0,T]; H^\delta)$ *for* $\delta \in (0, 1-\beta)$. *Moreover, the mapping*

$$\mathcal{S} : H^{2-\beta} \times L^2([0,T]; H_+^{-\beta}) \ni (v, \mathcal{X}) \mapsto \mathcal{Y} \in L^2([0,T]; H^{1+\delta}) \cap C([0,T]; H^\delta)$$

is continuous.

In this talk we call $\Phi := Y + X$ the strong solution to (1), where Y is the solution appeared in Theorem 1 and X is the Ornstein-Uhlenbeck process. We are able to show that the strong solution corresponds to the limit of the stationary processes associated to approximating equations to (1).

Theorem 2. *Let* $|\alpha| < \sqrt{4\pi}$, $\varepsilon > 0$, *and* P_N *as above. Let* $N \in \mathbb{N}$ *and consider the solution* $\bar{\Phi}^N$ *to*

$$\begin{cases} \partial_t \bar{\Phi}_t^N = \dfrac{1}{2}(\triangle - 1)\bar{\Phi}_t^N - \dfrac{\alpha}{2} P_N \exp\left(\alpha P_N \bar{\Phi}_t^N - \dfrac{\alpha^2}{2} C_N\right) + \dot{W}_t, \\ \bar{\Phi}_0^N = \xi_N \in \mathcal{D}'(\Lambda) \end{cases} \tag{3}$$

where ξ_N *be a random variable with the law* $\mu_N^{(\alpha)}$ *and independent of* W. *Then* $\bar{\Phi}^N$ *is a stationary process and converges in law to the strong solution with an initial law* $\mu_{\exp}^{(\alpha)}$, *in the space* $C([0,T]; H^{-\varepsilon}(\Lambda))$ *for any* $T > 0$. *Moreover, the law of* $\bar{\mathbf{\Phi}}_t$ *is* $\mu_{\exp}^{(\alpha)}$ *for any* $t \geq 0$.

In [1], the Markov process $\mathbb{M} = (\Theta, \mathcal{G}, (\mathcal{G}_t)_{t\geq 0}, (\Psi_t)_{t\geq 0}, (Q_\phi)_{\phi\in E})$ associated to (1) by the Dirichlet form theory. As follows, we obtain the identification between the processes obtained in Theorem 1 and [1].

Theorem 3. *Let* $|\alpha| < \sqrt{4\pi}$. *Then for* $\mu_{\exp}^{(\alpha)}$-*a.e.* ϕ, *the diffusion process* Ψ *coincides with the strong solution* Φ *of (1) driven by some* $L^2(\Lambda)$-*cylindrical* $(\mathcal{G}_t)$-*Brownian motion* $\mathcal{W} = (\mathcal{W}_t)_{t\geq 0}$ *with the initial value* ϕ, Q_ϕ-*almost surely.*

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Applications of non-local Dirichlet forms defined on infinite dimensional spaces.

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Abstract

By [AKYY 2019] (with detailed proofs), [AK 2019] (with a concise explanation), [Sympo.2019] and [Sympo.2018], general theorems on the closability and quasi-regularity of non-local Markovian symmetric forms on probability spaces $(S, \mathcal{B}(S), \mu)$, with S Fréchet spaces such that $S \subset \mathbb{R}^{\mathbb{N}}$, $\mathcal{B}(S)$ is the Borel σ-field of S, and μ is a Borel probability measure on S, have been introduced. There, a family of non-local Markovian symmetric forms $\mathcal{E}_{(\alpha)}$, $0 < \alpha < 2$, acting in each given $L^2(S;\mu)$ was defined, the index α characterizing the order of the non-locality. Then, it has been shown that all the forms $\mathcal{E}_{(\alpha)}$ defined on $\bigcup_{n\in\mathbb{N}} C_0^\infty(\mathbb{R}^n)$ are closable in $L^2(S;\mu)$. Moreover, sufficient conditions under which the closure of the closable forms, that are Dirichlet forms, become strictly quasi-regular, has been given. Also, an existence theorem for Hunt processes properly associated to the Dirichlet forms has been introduced. In addition, the application of the above abstract theorems to the problem of stochastic quantizations of Euclidean Φ_d^4 fields, for $d = 2, 3$, by means of these Hunt processes has been indicated.

In the present talk, we shall introduce several applications of the abstract theorem to the stochastic quantizations, in the non-local sense, of various random fields:

1. Non-local type stochastic quantization of Euclidean $P(\Phi)_2$ fields.
We consider a *non-local* type stochastic quantization of the *finite* volume Euclidean $P(\Phi)_2$ field.

2. Non-local type stochastic quantization of Euclidean quantum field with exponential potential.
We consider a *non-local* type stochastic quantization of the 2-dimensional Euclidean quantum field with *finite* volume exponential potential.

3. Non-local type stochastic quantization of Euclidean quantum field with trigonometric potentials.
We consider a *non-local* type stochastic quantization of the 2-dimensional Euclidean quantum field with *finite* volume trigonometric potentials.

4. Non-local type stochastic quantization of a field of classical infinite particle system.

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Implicit Euler–Maruyama scheme for radial Dunkl processes

Dai Taguchi (Okayama University)
joint work with
Hoang-Long Ngo (Hanoi National University of Education)

Abstract

Let R be a (reduced) root system in $\mathbb{R}^d$ and W be the associated reflection group. For given a vector $\xi \in \mathbb{R}^d$, the Dunkl operator T_ξ on $\mathbb{R}^d$ associated with W are introduced by Dunkl [4] and are differential-difference operators given by

$$T_\xi f(x) := \frac{\partial f(x)}{\partial \xi} + \sum_{\alpha \in R_+} k(\alpha) \langle \alpha, \xi \rangle \frac{f(x) - f(\sigma_\alpha x)}{\langle \alpha, x \rangle},$$

where $\frac{\partial}{\partial \xi}$ is the directional derivative with respect to ξ, and σ_α is the orthogonal reflection with respect to $\alpha \in \mathbb{R}^d \setminus \{0\}$, R_+ is a positive subsystem of the root system R and $k : R \to [1/2, \infty)$ is a multiplicity function. Dunkl operators have been widely studied in both mathematics and physics, for example, there operators play a crucial role to the study special functions associated with root systems and the Hamiltonian operators of some Calogero-Moser-Sutherland quantum mechanical systems. Moreover, the Dunkl Laplacian defined by $\Delta_k f(x) := \sum_{i=1}^d T_{\xi_i}^2$, for any orthonormal basis $\{\xi_1, \ldots, \xi_d\}$ of $\mathbb{R}^d$ is an important, and it has the following explicit form

$$\Delta_k f(x) = \Delta f(x) + 2 \sum_{\alpha \in R_+} k(\alpha) \left\{ \frac{\langle \nabla f(x), \alpha \rangle}{\langle \alpha, x \rangle} + \frac{f(\sigma_\alpha x) - f(x)}{\langle \alpha, x \rangle^2} \right\}.$$

Rösler [7] studied Dunkl heat equation $(\Delta_k - \partial_t)u,\ u(\cdot, 0) = f \in C_b(\mathbb{R}^d; \mathbb{R})$ and Rösler and Voit [8] introduced Dunkl processes Y which are càdlàg Markov processes with infinitesimal generator $\Delta_k/2$ and is martingale with the scaling property. On the other hand, a radian Dunkl process $X = (X(t))_{t \geq 0}$ is a continuous Markov process with infinitesimal generator $L_k^W/2$ defined by

$$\frac{L_k^W f(x)}{2} := \frac{\Delta f(x)}{2} + \sum_{\alpha \in R_+} k(\alpha) \frac{\langle \nabla f(x), \alpha \rangle}{\langle \alpha, x \rangle},$$

and is W-radial part of the Dunkl process Y, that is, for the canonical projection $\pi : \mathbb{R}^d \to \mathbb{R}^d/W$, $X = \pi(Y)$, as identifying the space $\mathbb{R}^d/W$ to (fundamental) Weyl chamber $\mathbb{W} := \{x \in \mathbb{R}^d\ ;\ \langle \alpha, x \rangle > 0,\ \alpha \in R_+\}$ of the root system R. Schapira [9] and Demini [2] proved that a radial Dunkl process X satisfies the following $\mathbb{W}$-valued stochastic differential equation (SDE)

$$\mathrm{d}X(t) = \mathrm{d}B(t) + \sum_{\alpha \in R_+} k(\alpha) \frac{\alpha}{\langle \alpha, X(t) \rangle} \mathrm{d}t,\ X(0) = x(0) \in \mathbb{W}, \tag{1}$$

where $B = (B(t))_{t\geq 0}$ is a d-dimensional standard Brownian motion. For example, if $R := \{\pm 1\}$ then X is a Bessel process, and for type A_{d-1} root system, that is, $R := \{e_i - e_j \in \mathbb{R}^d \ ; \ i \neq j\} \subset \{x \in \mathbb{R}^d; \sum_{i=1}^d x_i = 0\}$, then X is a Dyson's Brownian motion.

In this talks, inspired by [1, 3, 5, 6], we study a numerical analysis for radial Dunkl processes corresponding to arbitrary (reduced) root systems in $\mathbb{R}^d$, not only Bessel processes and Dyson's Brownian motions. We introduce an implicit Euler–Maruyama scheme for radial Dunkl processes (1), which takes values in the domain Wely chamber $\mathbb{W}$, and provide its rate of convergence in L^p-sup norm and path-wise sense. The key idea of the proof is to use the change of measure based on Girsanov theorem for radial Dunkl processes, which was proved in [10] for the Bessel case.

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Parametrix method for multi-skewed Brownian motion

Gô Yûki (Ritsumeikan University)

Let $x_0 \in \mathbf{R}$, $n \in \mathbf{N}$, $\alpha_1, \ldots, \alpha_n \in (-1, 1)$ and $-\infty < a_1 < a_2 < \cdots < a_n < +\infty$. We consider one dimensional SDEs of the form

$$X_t(x_0) = x_0 + B_t + \sum_{i=1}^{n} \alpha_i L_t^{a_i}(X), \tag{1}$$

where $\{B_t\}_{t \geq 0}$ is a one-dimensional Brownian motion and $L_t^{a_i}(X)$ denotes the symmetric local time of X at the point a_i until the time t. If $n = 1$ and $a_1 = 0$, the process X is called the skew Brownian motion. In [3], one can find exact simulation methods for the skew Brownian motion. These methods have been extended to some other cases in [1] using resolvent methods. In the case of $n = 2$, a simulation method has been proposed in [2] which points out at the difficulty of obtaining exact simulations methods for $n \geq 3$. In this talk, we propose a simulation method for any n. The method is based on an expansion for $\mathbf{E}[f(X_t(x_0))]$ which is obtained by the parametric method.

Main Result

Let us define $b_i := \frac{a_i + a_{i+1}}{2}$ for $i = 1, \cdots, n-1$. Fix $0 < \varepsilon < \min_{1 \leq i \leq n-1} \{\frac{a_{i+1} - a_i}{2}\}$ arbitrary. Let $\varphi_1, \ldots, \varphi_n$ be elements of $C_b^2(\mathbf{R})$ which satisfy the following conditions.

(H1): $\sum_{i=1}^{n} \varphi_i \equiv 1$.

(H2): $\operatorname{supp} \varphi_1 = (-\infty, b_1 + \varepsilon]$, $\varphi_1 = 1$ on $(-\infty, b_1 - \varepsilon]$ and decreasing on $(b_1 - \varepsilon, b_1 + \varepsilon]$.

(H3): For $2 \leq i \leq n-1$, $\operatorname{supp} \varphi_i = [b_{i-1} - \varepsilon, b_i + \varepsilon]$, increasing on $(b_{i-1} - \varepsilon, b_{i-1} + \varepsilon]$, $\varphi_i = 1$ on $[b_{i-1} + \varepsilon, b_i - \varepsilon]$ and decreasing on $(b_i - \varepsilon, b_i + \varepsilon]$.

(H4): $\operatorname{supp}\varphi_n = [b_{n-1}-\varepsilon, +\infty)$, $\varphi_n = 1$ on $[b_{n-1}+\varepsilon, +\infty)$ and increasing on $[b_{n-1}-\varepsilon, b_{n-1}+\varepsilon)$.

Let $\widetilde{X}^i(x_0)$ be the solution to the SDE

$$\widetilde{X}^i_t(x_0) = x_0 + B_t + \alpha_i L^{a_i}_t(\widetilde{X}^i(x_0)). \tag{2}$$

For a bounded Borel measurable function f, we put

$$P_t f(x) := \mathbf{E}[f(X_t(x))] \text{ and } \widetilde{P}_t f(x) := \sum_{i=1}^{n} \mathbf{E}[\varphi_i(\widetilde{X}^i_t(x)) f(\widetilde{X}^i_t(x))] \ (t \in (0,T]).$$

Theorem 0.1. *Let $x_0 \in \mathbf{R}$, X be a solution to* (1) *and for $i = 1, \cdots, n$, $\widetilde{X}^i$ be a solution to* (2)*. Then for $t \in (0,T]$, we have that*

(3)

$$P_t f(x_0) = \widetilde{P}_t f(x_0) + \sum_{i=1}^{n} \int_0^t \mathbf{E}\left[\widetilde{P}_{t-s} f(\widetilde{X}^i_s(x_0)) \Theta^i_s(x_0, \widetilde{X}^i_s(x_0))\right] ds$$
$$+ \sum_{m=2}^{\infty} \sum_{1 \le i_1, \cdots, i_m \le n} \int_{\Delta_m(t)} \mathbf{E}\left[\widetilde{P}_{t-\sum_{l=1}^m s_l} f(\widetilde{Y}^{i_m}_{s_m}) \prod_{j=1}^{m} \Theta^{i_j}_{s_j}(\widetilde{Y}^{i_{j-1}}_{s_{j-1}}, \widetilde{Y}^{i_j}_{s_j})\right] \mathbf{ds^m_1},$$

where $\widetilde{Y}^{i_0}_{s_0} := x_0$, $\widetilde{Y}^{i_j}_{s_j} := \widetilde{X}^{i_j}_{s_j}(\widetilde{Y}^{i_{j-1}}_{s_{j-1}})$ for $j \ge 1$, $\Theta^i_s(x_0, x) := \frac{1}{2}\left(\varphi''_i(x) + 2\varphi'_i(x)\frac{\partial_x p^i_s(x_0,x)}{p^i_s(x_0,x)}\right)$ and p^i denotes the transition density function of $\widetilde{X}^i$.

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Stochastic flows and rough differential equations on foliated spaces

Yuzuru INAHAMA (Kyushu Univ.)

In this talk we construct stochastic flows associated with SDEs on compact foliated spaces via rough path theory.

In 2015 Suzaki constructed "leafwise diffusion processes"on compact foliated spaces via SDE theory. However, it is not known whether the stochastic flows associated to them exist or not. The main difficulty is in showing the existence of continuous modifications. The reason is that Kolmogorov-Centsov criterion is not available in this case since a foliated space is just a locally compact metric space.

From the viewpoint of rough path theory, however, there is in fact not much difficulty here and this problem is naturally and easily solved.

Since stochastic flows play a very important role in stochastic analysis on manifolds, we hope our result would open the door for stochastic analysis on foliated spaces.

This is a joint work with Kiyotaka SUZAKI (Kumamoto Univ.) and can be found at arXiv:1910.09962.