Molchanov's technique for small-time heat kernel asymptotics at cut points

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We discuss a technique, going back to work of Molchanov, for determining the small-time asymptotics of the heat kernel (or equivalently, a type of large deviation of the corresponding diffusion) at the cut locus of a sub-Riemannian manifold (valid away from any abnormal geodesics). We relate the leading term of the expansion to the structure of the cut locus, especially to the conjugacy of the minimizing geodesics. In general, one can determine bounds on the leading power of t, and the more one assumes about the local structure of the minimizing geodesics (or the exponential map), the more precise these bounds become. Taking this line of reasoning further, the exact power of t can be determined in a variety of cases, including generic Riemannian and sub-Riemannian metrics in low dimension (and one consequence is that we obtain restrictions on the types of singularities of the exponential map that can, generically, occur along minimizing geodesics) and metrics that exhibit rotational symmetries.

Moreover, we discuss the asymptotics for the gradient and Hessian of the logarithm of the heat kernel on a Riemannian manifold, giving a characterization of the cut locus in terms of the behavior of the log-Hessian. In particular, the leading term in the expansion of the log-Hessian comes from the "variance of the minimizing geodesics" with respect to the small-time Brownian bridge.

This work is joint with Davide Barilari, Ugo Boscain, and Grégoire Charlot.