

Random walks, sub-Laplacians, and volumes in sub-Riemannian geometry

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We study a variety of random walks on sub-Riemannian manifolds and their diffusion limits under parabolic scaling, which give, via their infinitesimal generators, second-order operators on the manifolds. A primary motivation is the lack of a canonical sub-Laplacian in sub-Riemannian geometry, and thus we are particularly interested in the relationship between the infinitesimal generators, the geodesic structure, and operators which can be obtained as divergences with respect to various choices of volume. In particular, we give a general criterion for the divergence of the gradient with respect to a smooth volume to also be realized as the infinitesimal generator coming from a horizontal random walk. We then determine how this turns out for some important classes of sub-Riemannian structures, such as the contact case (where things work out nicely) and Carnot groups (where one cannot expect uniqueness), as well as a “pathological” example of a 4-dimensional sub-Riemannian structure where there is a canonical choice of volume but the corresponding operator is not realized via a horizontal random walk.

On the probabilistic side, we give a general result on the convergence of random walks to a diffusion on a sub-Riemannian manifold, which is in the spirit of earlier work of Stroock-Varadhan on Euclidean space.

This work is joint with Ugo Boscain, Luca Rizzi, and Andrei Agrachev.