Paracontrolled quasi-geostrophic equation with space-time white noise

Yuzuru INAHAMA *

This is a joint work with Yoshihiro SAWANO (Tokyo Metropolitan University). The preprint is uploaded on arXiv Preprint Server (arXiv:1806.00564).

In this talk we discuss the stochastic dissipative quasi-geostrophic equation (QGE for short) with space-time white noise on the two-dimensional torus $\mathbf{T}^2$. This equation is highly singular and basically ill-posed in its original form. The main objective of the present talk is to formulate and solve this equation locally in time in the framework of paracontrolled calculus when the differential order of the main term, the fractional Laplacian, is larger than 7/4. No renormalization has to be done for this model.

The dissipative QGE is a partial differential equation which describes geophysical fluid dynamics on a two-dimensional space.

We study the following stochastic dissipative QGE with *additive* space-time white noise ξ on $\mathbf{T}^2$ with a given initial condition:

$$\partial_t u = -(-\Delta)^{\theta/2} u + R^{\perp} u \cdot \nabla u + \xi, \qquad t > 0, \ x \in \mathbf{T}^2. \tag{1}$$

Here, (i) $\nabla = (\partial_1, \partial_2)$ is the usual gradient on $\mathbf{T}^2$, (ii) $R^{\perp} = (R_2, -R_1)$ with R_j being the jth Riesz transform on $\mathbf{T}^2$ ($j = 1, 2$), (iii) the dot stands for the standard inner product on $\mathbf{R}^2$, (iv) $\xi = \xi(t, x)$ is a (generalized) centered Gaussian random field associated with $L^2(\mathbf{R} \times \mathbf{T}^2)$ whose covariance is heuristically given by $\mathbb{E}[\xi(t, x)\xi(s, y)] = \delta(t - s)\delta(x - y)$, where δ stands for Dirac's delta function at 0.

Unlike regularity structure theory, there is no "bible" for paracontrolled calculus, since it has been developed gradually. For the probabilistic part, we follow Gubinelli-Perkowski's method (2017, CMP). For the deterministic part, we follow Mourrat-Weber's method (2017+, AoP).

*Kyushu University, Japan. Email:inahama@math.kyushu-u.ac.jp