Parisian reflected Lévy processes

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1. Parisian-reflected Lévy processes

Let $X = (X(t); t \geq 0)$ be a spectrally negative Lévy process defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with its Laplace exponent $\psi(\theta) : [0, \infty) \to \mathbb{R}$, i.e. $\mathbb{E}\big[\mathrm{e}^{\theta X(t)}\big] =: \mathrm{e}^{\psi(\theta)t}$, $t, \theta \geq 0$, given by the *Lévy-Khintchine formula*

$$\psi(\theta) := \gamma\theta + \frac{\sigma^2}{2}\theta^2 + \int_{(-\infty,0)} \big(\mathrm{e}^{\theta x} - 1 - \theta x \mathbf{1}_{\{x>-1\}}\big)\Pi(\mathrm{d}x), \quad \theta \geq 0,$$

where $\gamma \in \mathbb{R}$, $\sigma \geq 0$, and Π is a measure on $(-\infty, 0)$ known as the Lévy measure of X that satisfies $\int_{(-\infty,0)}(1 \wedge x^2)\Pi(\mathrm{d}x) < \infty$. In addition, let $\mathcal{T}_r = \{T(i); i \in \mathbb{N}\}$ be an increasing sequence of epochs of a Poisson process with rate $r > 0$, independent of X.

We construct the *Lévy process with Parisian reflection below* $X_r = (X_r(t); t \geq 0)$ as follows: the process is only observed at times $\mathcal{T}_r$ and is pushed up to 0 if and only if it is below 0. More precisely, we have

$$X_r(t) = X(t), \quad 0 \leq t < T_0^-(1) \tag{1}$$

where

$$T_0^-(1) := \inf\{S \in \mathcal{T}_r : X(S-) < 0\}. \tag{2}$$

The process is then pushed upward by $|X(T_0^-(1))|$ so that $X_r(T_0^-(1)) = 0$. For $T_0^-(1) \leq t < T_0^-(2) := \inf\{S \in \mathcal{T}_r : S > T_0^-(1), X_r(S-) < 0\}$, we have $X_r(t) = X(t) + |X(T_0^-(1))|$. The process can be constructed by repeating this procedure.

Suppose $R_r(t)$ is the cumulative amount of (Parisian) reflection until time $t \geq 0$. Then we have

$$X_r(t) = X(t) + R_r(t), \quad t \geq 0,$$

with

$$R_r(t) := \sum_{i=1}^{\infty} 1_{\{T_0^-(i) \leq t\}} |X_r(T_0^-(i)-)|, \quad t \geq 0, \tag{3}$$

where $(T_0^-(n); n \geq 1)$ can be constructed inductively by (2) and

$$T_0^-(n+1) := \inf\{S \in \mathcal{T}_r : S > T_0^-(n), X_r(S-) < 0\}, \quad n \geq 1.$$

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2. Fluctuation identities

Define

$$\tau_a^-(r) := \inf\{t > 0 : X_r(t) < a\} \quad \text{and} \quad \tau_a^+(r) := \inf\{t > 0 : X_r(t) > a\}, \quad a \in \mathbb{R}.$$

We obtain several fluctuation identities including the following:

1. Joint Laplace transform with killing: for all $q, \theta \geq 0$, $a < 0 < b$, and $x \leq b$,

$$g(x,a,b,\theta) := \mathbb{E}_x\left(e^{-q\tau_b^+(r)-\theta R_r(\tau_b^+(r))}; \tau_b^+(r) < \tau_a^-(r)\right),$$
$$h(x,a,b,\theta) := \mathbb{E}_x\left(e^{-q\tau_a^-(r)-\theta R_r(\tau_a^-(r))}; \tau_a^-(r) < \tau_b^+(r)\right).$$

2. Total discounted values of Parisian reflection: for $a < 0 < b$, $q \geq 0$, and $x \leq b$,

$$f(x,a,b) := \mathbb{E}_x\left(\int_0^{\tau_b^+(r)\wedge\tau_a^-(r)} e^{-qt}\mathrm{d}R_r(t)\right).$$

These can be written in terms of the scale function of the spectrally negative Lévy process.

In the talk, several extensions/modifications of this process are also discussed, including the cases with additional classical reflection from above/below and also the cases X is replaced with a spectrally positive Lévy process.

3. Applications in Insurance

In de Finett's optimal dividend problem, one wants to choose the optimal dividend policy so as to maximize the total expected value of discounted dividends accumulated until ruin.

Consider its version where dividend payments can be made only at the jump times of an independent Poisson process, *a Parisian reflection strategy* is expected to be optimal. Namely, given a suitable barrier b^*, it is optimal to pay dividends at each dividend payment decision time if and only if the surplus is above b^* – the resulting surplus process becomes a Parisian-reflected process. The optimality is shown in [2] for the spectrally negative case with a completely monotone Lévy density, and in [4] for the spectrally positive case.

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