A RELATION BETWEEN MODELED DISTRIBUTIONS AND PARACONTROLLED DISTRIBUTIONS

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In the field of singular SPDEs, there are two big theories: the theory of *regularity structures* [4] by Hairer and the *paracontrolled calculus* [2] by Gubinelli, Imkeller and Perkowski. These two theories are based on a common principle but composed of different mathematical tools. Therefore we can use either of them according to the situation. For example, the former is useful to show a universal property of a large number of SPDEs (e.g. [5, 6]), and the latter is useful to get more detailed information of a specific SPDE (e.g. [3, 7]). However, there is a gap between the two theories about the range of application. For example, the Hairer's theory can be applied to the 3-dimensional parabolic Anderson model

$$(\partial_t - \Delta)u(t,x) = u(t,x)\xi(x), \quad t > 0,\ x \in \mathbb{T}^3,$$

for $\xi \in \mathcal{C}^{-3/2-\epsilon}(\mathbb{T}^3)$ with $\epsilon > 0$, but the GIP theory cannot be.

In this talk, we discuss how to overcome this gap. Recently, Bailleul and Bernicot [1] are tying to improve the GIP theory. Our plan is to complete their work by combining the essence of the Hairer's theory. There is a difference between both theories about the definition of solutions. In the Hairer's theory, the solution is defined as a *modeled distribution*, which represents a local behavior of the solution. In the GIP theory, the solution is defined as a *paracontrolled distribution*, which is defined by nonlocal operators. Each definition has an advantage to each other. We compare these two notions and aim to find a better way.

References

[1] I. Bailleul and F. Bernicot, *Higher order paracontrolled calculus*, arXiv: 1609.06966.

[2] M. Gubinelli, P. Imkeller, and N. Perkowski, *Paracontrolled distributions and singular PDEs*, Forum Math. Pi **3** (2015), e6, 75pp.

[3] M. Gubinelli and N. Perkowski, *KPZ reloaded*, Comm. Math. Phys. **349** (2017), no. 1, 165-269.

[4] M. Hairer, *A theory of regularity structures*, Invent. Math. **198** (2014), no. 2, 269-504.

[5] M. Hairer and J. Mattingly, *The strong Feller property for singular stochastic PDEs*, arXiv: 1610.03415.

[6] M. Hairer and J. Quastel, *A class of growth models rescaling to KPZ*, arXiv: 1512.07845.

[7] J.-C. Mourrat and H. Weber, *The dynamic Φ^4_3 model comes down from infinity*, arXiv: 1601.01234.