Identification of finite variation processes from the SFCs

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1. Introduction

It has been discussed in [1]-[6] and [8] the question whether a random function (or a stochastic derivative as an extension) is identified from its stochastic Fourier coefficients (SFCs). In the previous studies mentioned above, affirmative answers to this question are given. In [1] and [2], the random function is causal. In [5] and [6], the random function is noncausal and absolutely continuous and the SFC is given by the Ogawa integral. In this talk, we show any finite variation process (or the stochastic differential as an extension) is identified from its SFCs of Ogawa type with respect to any CONS of $L^2[0,1]$. We also show identification on infinite time interval and identification from SFC of Skorokhod type.

2. Setting

Let $(B_t)_{t\in[0,\infty)}$ be a Brownian motion on a probability space $(\Omega, \mathcal{F}, P)$. By the symbol $[0,L]$, we mean the finite closed interval from 0 to L if $0 < L < \infty$, and $[0,\infty)$ if $L = \infty$. Let $(e_i)_{i\in\mathbb{N}}$ be a CONS of $L^2[0,L]$. We denote the Ogawa integral by $\int_0^L \, d_{\mathrm{u}}B$, the Sobolev space by $\mathcal{L}_i^{r,2}$ and the Skorokhod integral by $\int_0^L \, dB$ (see Definition 1,2 and 4 of [7]).

Hereafter we consider measurable maps, what we call random functions, $a, b : [0,L] \times \Omega \to \mathbb{C}$ such that $b \in L^2[0,L]$ a.s.

Definition 1 (SFC-O of stochastic differential) Suppose ae_i is Ogawa integrable for every $i \in \mathbb{N}$. We define the SFC of Ogawa type (SFC-O) $(d_{\mathrm{u}}Y, e_i)$ of the stochastic differential $d_{\mathrm{u}}Y_t = a(t)\, d_{\mathrm{u}}B_t + b(t)\, dt$, $t \in [0,L]$ with respect to $(e_i)_{i\in\mathbb{N}}$ by

$$(d_{\mathrm{u}}Y, e_i) := \int_0^L \overline{e_i(t)}\, d_{\mathrm{u}}Y_t = \int_0^L a(t)\overline{e_i(t)}\, d_{\mathrm{u}}B_t + \int_0^L b(t)\overline{e_i(t)}\, dt.$$

Definition 2 (SFC-S of stochastic differential) Suppose $ae_i \in \mathcal{L}_1^{1,2}$ for every $i \in \mathbb{N}$. We define the SFC of Skorokhod type (SFC-S) (dX, e_i) of the stochastic differential $dX_t = a(t)\, dB_t + b(t)\, dt$, $t \in [0,L]$ with respect to $(e_i)_{i\in\mathbb{N}}$ by

$$(dX, e_i) := \int_0^L \overline{e_i(t)}\, dX_t = \int_0^L a(t)\overline{e_i(t)}\, dB_t + \int_0^L b(t)\overline{e_i(t)}\, dt.$$

3. Main Theorems

Theorem 1 (Identification from SFCs-O of stochastic differential) Assume a is any real finite variation process. a and b are identified from the system of SFCs-O $((d_{\mathrm{u}}Y, e_i))_{i\in\mathbb{N}}$ of the stochastic differential $d_{\mathrm{u}}Y_t = a(t)\, d_{\mathrm{u}}B_t + b(t)\, dt$.

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Remark 1 If $a \geq 0$ $\lambda \otimes P$-a.e. , then a is identified in the strong sense , i.e. identified from only SFCs without supplementary information such as $(B_t)_{t\in[0,L]}$.
Remark 2 a is identified as a member of $L^0([0,L]\times\Omega)$, but if a is assumed to be left continuous, a is specified for every $t \in [0,L]$ almost surely.
Remark 3 Even if the system of SFCs lacks its finite elements $(d_{\mathrm{u}}Y, e_i)$, a can be identified.

Proposition 1 (Ogawa integral of H-S integral transform of Wiener functional) Let $f \in \mathcal{L}_1^{1,2}$ and $K \in L^2([0,L]^2)$. Then, $F(t) = \int_0^L K(t,s)f(s)\,ds$ is u-integrable and the Ogawa integral is given by

$$\int_0^L F(t)\,d_{\mathrm{u}}B_t = \int_0^L F(t)\,dB_t + \int_0^L\int_0^L K(t,s)D_tf(s)\,ds\,dt \quad \text{in } L^2(\Omega).$$

Theorem 2 (Identification from SFCs-S of stochastic differential) Assume a satisfies the following:

(1) $a(t)$ is real local absolutely continuous a.s.

(2) $a'(t) \in \mathcal{L}_1^{1,2}$, $a(0) \in \mathcal{L}_0^{1,2}$.

(3) $a'(t) \in L^1[0,L]$ a.s. , $\int_0^t D_t a'(s)\,ds \in L^2[0,L]$ a.s.

Then, a and b are identified from the system of SFCs-S $((dX, e_i))_{i\in\mathbb{N}}$ of the stochastic differential $dX_t = a(t)\,dB_t + b(t)\,dt$.

Remark 1 If $a \geq 0$ $\lambda \otimes P$-a.e. , then a is identified in the strong sense.
Remark 2 a is specified for every $t \in [0,L]$ almost surely.
Remark 3 Even if the system of SFCs lacks its finite elements (dX, e_i), a can be identified.
Remark 4 If $L < \infty$, then (3) holds.

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