2017年度　確率解析とその周辺

(Stochastic Analysis and Related Topics 2017)

予稿集

(Abstracts)

2017年 10月16日(月) 14:00 〜 10月18日(水) 16:20

(Oct. 16 (Mon.) – Oct. 18 (Wed.), 2017)

立命館大学びわこ・くさつキャンパス（BKC)
ウエストウィング6階　談話会室

(Colloquium Room, West Wing 6F,
Biwako-Kusatsu Campus (BKC),
Ritsumeikan University)

—プログラム—

10 月 16 日 (月)

14:00〜14:40　重川 一郎 (Ichiro Shigekawa)　(京都大学)

Kolmogorov-Pearson diffusions and hypergeometric functions

14:50〜15:30　星野 浄生 (Kiyoiki Hoshino)　(大阪府立大学)

Identification of finite variation processes from the SFCs

15:50〜16:30　廣島 文生 (Fumio Hiroshima)　(九州大学)

Renormalized Gibbs measures and applications to QFT

16:40〜17:20　山崎 和俊 (Kazutoshi Yamazaki)　(関西大学)

Parisian reflected Lévy processes

10 月 17 日 (火)

10:00〜10:40　星野 壮登 (Masato Hoshino)　(早稲田大学)

A relation between modeled distributions and paracontrolled distributions

10:50〜11:30　楠岡 誠一郎 (Seiichiro Kusuoka)　(岡山大学)

The invariant measure and flow associated to the Φ^4_3-quantum field model

11:40〜12:40　Roland Friedrich （Saarland University）　[特別講演]

Operads and Stochastic Calculus

12:40〜14:00　昼休み

14:00〜14:40　難波 隆弥 (Ryuya Namba)　(岡山大学)

A Functional central limit theorem for non-symmetric random walks on step-r nilpotent covering graphs

14:50〜15:30　赤堀 次郎 (Jiro Akahori)　(立命館大学)

Supersymmetry in Wiener Space

15:50〜16:30　福島 正俊 (Masatoshi Fukushima)　(大阪大学名誉教授)

Free fields and extended Dirichlet spaces

16:40〜17:20　村山 拓也 (Takuya Murayama)　(京都大学)

Characterization of the explosion time for the Komatu–Loewner evolution

10月18日(水)

10:00〜10:40　天羽 隆史 (Takafumi Amaba)　(福岡大学)

Distributional Itô's formula and regularization of generalized Wiener functionals

10:50〜11:30　世良 透 (Toru Sera)　(京都大学)

Multiray generalization of the arcsine laws for occupation times of infinite ergodic transformations

11:40〜12:40　Roland Friedrich （Saarland University）　[特別講演]

Controlled Kufarev-Loewner equations and the Sato-Segal-Wilson Grassmannian

12:40〜14:00　昼休み

14:00〜14:40　浜口 雄史 (Yushi Hamaguchi)　(京都大学)

Arbitrage theory in large financial markets

14:50〜15:30　田口 大 (Dai Taguchi)　(大阪大学)

Discrete approximations for non-colliding SDEs

15:40〜16:20　稲浜 譲 (Yuzuru Inahama)　(九州大学)

Heat trace asymptotics for equiregular sub-Riemannian manifolds
(with Setsuo Taniguchi)

Kolmogorov-Pearson diffusions and hypergeometric functions

Ichiro SHIGEKAWA (Kyoto University)

1 Introduction

We consider diffusuions generated by $\mathfrak{A} = a\frac{d^2}{dx^2} + b\frac{d}{dx}$. Here a is a quadratic function and b is a linear function. We call these diffusions as Kolmogorof-Pearson diffusions. We are interested in spectra of these generators. We want to determin all spectra completely. To do this, hypergeometric functions play a important role.

2 Sevral expressions of generators

Our generators are of the form

$$\mathfrak{A} = a\frac{d^2}{dx^2} + b\frac{d}{dx} \tag{1}$$

where a is quadtatic and b is linear. Following Feller, we can associate a measure dm and a function s. dm is called a speed measure and s is called a scale function. In our case, dm has a density ρ of the form $\rho = \exp\{\int (f/g)dx\}$ where f is linear and g is quadratic. We call this type of density as Pearson density. Pearson considered probability densities but we may admit infinite measure cases. s defines a measure ds and it has of the form $ds = \dfrac{1}{a\rho}dx$. Using a and ρ, b can be expressed as $b = a' + a(\log\rho)'$.

Now we can give several expressions of the generator as follows:

	generator	duality	differential opetaor
Kolmogorov	$a\dfrac{d^2}{dx^2} + b\dfrac{d}{dx}$		
Feller	$\dfrac{d}{dm}\dfrac{d}{ds}$	$\dfrac{d}{dm} = -\dfrac{d}{ds}^*$	$\dfrac{d}{ds} : L^2(dm) \to L^2(ds)$
Stein	$\left(a\dfrac{d}{dx} + b\right)\dfrac{d}{dx}$	$a\dfrac{d}{dx} + b = -\dfrac{d}{dx}^*$	$\dfrac{d}{dx} : L^2(\rho dx) \to L^2(a\rho dx)$

Using this, we can make following correspondences.

Feller's pair	$\dfrac{d}{dm}\dfrac{d}{ds} \longleftrightarrow \dfrac{d}{dm}\dfrac{d}{ds}$
Stein's pair	$(a\dfrac{d}{dx} + b)\dfrac{d}{dx} \longleftrightarrow \dfrac{d}{dx}(a\dfrac{d}{dx} + b)$

One important thing is that the class of Kolmogorov-Pearson diffusions are closed under Feller's pair and Stein's pair. From these pairings, we can show that

- If f is an eigenfunction, then so are f', $\frac{d}{ds}f$.
- If θ is an eigenfunction, then so are $a\theta' + b\theta$, $\frac{d}{dm}\theta$.

According to the degree of a, our generators are classified as

	complete family	incomplete family		special function
α-family	$a = 1$			F_1^0
β-family	$a = x$	$a = x^2$		F_1^1
γ-family	$a = x(1-x)$	$a = x(1+x)$	$a = 1 + x^2$	F_1^2

Further, associated speed measures are given as follows:

	complete family	incomplete family	
α-family	$e^{\beta x^2/2}$		
β-family	$x^\alpha e^{\beta x}$	$x^\alpha e^{\beta/x}$	
γ-family	$x^\alpha(1-x)^\beta$	$x^\alpha(1+x)^\beta$	$(1+x^2)^\alpha \exp\{2\beta \arctan x\}$

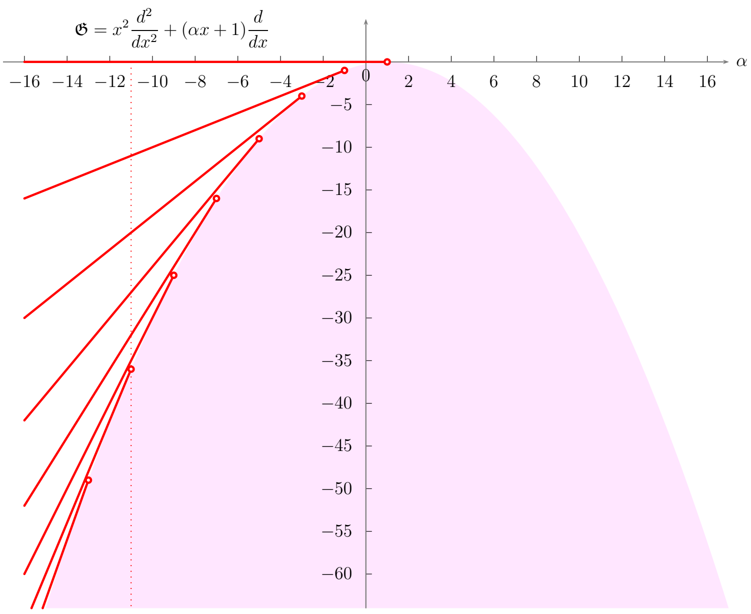
3 Spectra of generators

We have the following six cases:
(i) $a = 1$, (ii) $a = x$, (iii) $a = x^2$, (iv) $a = x(1-x)$, (v) $a = x(1+x)$, (vi) $a = 1 + x^2$.

We have discussed (i) and (ii) in the previous occasion. We will discuss here (iii) – (vi). In the case of (iii), the generator has the following form:

$$\mathfrak{A} = x^2 \frac{d}{dx^2} + (\alpha x - \beta)\frac{d}{dx}. \tag{2}$$

In particular, in the case $\beta = -1$, spectra are given as



Other cases will be discussed in the talk.

Identification of finite variation processes from the SFCs

Kiyoiki Hoshino (Osaka prefecture university)*

1. Introduction

It has been discussed in [1]-[6] and [8] the question whether a random function (or a stochastic derivative as an extension) is identified from its stochastic Fourier coefficients (SFCs). In the previous studies mentioned above, affirmative answers to this question are given. In [1] and [2], the random function is causal. In [5] and [6], the random function is noncausal and absolutely continuous and the SFC is given by the Ogawa integral. In this talk, we show any finite variation process (or the stochastic differential as an extension) is identified from its SFCs of Ogawa type with respect to any CONS of $L^2[0,1]$. We also show identification on infinite time interval and identification from SFC of Skorokhod type.

2. Setting

Let $(B_t)_{t\in[0,\infty)}$ be a Brownian motion on a probability space $(\Omega, \mathcal{F}, P)$. By the symbol $[0,L]$, we mean the finite closed interval from 0 to L if $0 < L < \infty$, and $[0,\infty)$ if $L = \infty$. Let $(e_i)_{i\in\mathbb{N}}$ be a CONS of $L^2[0,L]$. We denote the Ogawa integral by $\int_0^L d_\mathrm{u}B$, the Sobolev space by $\mathcal{L}_i^{r,2}$ and the Skorokhod integral by $\int_0^L dB$ (see Definition 1,2 and 4 of [7]).

Hereafter we consider measurable maps, what we call random functions, $a, b : [0,L]\times\Omega \to \mathbb{C}$ such that $b \in L^2[0,L]$ a.s.

Definition 1 (SFC-O of stochastic differential) Suppose ae_i is Ogawa integrable for every $i \in \mathbb{N}$. We define the SFC of Ogawa type (SFC-O) $(d_\mathrm{u}Y, e_i)$ of the stochastic differential $d_\mathrm{u}Y_t = a(t)\,d_\mathrm{u}B_t + b(t)\,dt$, $t \in [0,L]$ with respect to $(e_i)_{i\in\mathbb{N}}$ by

$$(d_\mathrm{u}Y, e_i) := \int_0^L \overline{e_i(t)}\,d_\mathrm{u}Y_t = \int_0^L a(t)\overline{e_i(t)}\,d_\mathrm{u}B_t + \int_0^L b(t)\overline{e_i(t)}\,dt.$$

Definition 2 (SFC-S of stochastic differential) Suppose $ae_i \in \mathcal{L}_1^{1,2}$ for every $i \in \mathbb{N}$. We define the SFC of Skorokhod type (SFC-S) (dX, e_i) of the stochastic differential $dX_t = a(t)\,dB_t + b(t)\,dt$, $t \in [0,L]$ with respect to $(e_i)_{i\in\mathbb{N}}$ by

$$(dX, e_i) := \int_0^L \overline{e_i(t)}\,dX_t = \int_0^L a(t)\overline{e_i(t)}\,dB_t + \int_0^L b(t)\overline{e_i(t)}\,dt.$$

3. Main Theorems

Theorem 1 (Identification from SFCs-O of stochastic differential) Assume a is any real finite variation process. a and b are identified from the system of SFCs-O $((d_\mathrm{u}Y, e_i))_{i\in\mathbb{N}}$ of the stochastic differential $d_\mathrm{u}Y_t = a(t)\,d_\mathrm{u}B_t + b(t)\,dt$.

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Remark 1 If $a \geq 0 \;\; \lambda \otimes P$-a.e. , then a is identified in the strong sense , i.e. identified from only SFCs without supplementary information such as $(B_t)_{t\in[0,L]}$.
Remark 2 a is identified as a member of $L^0([0,L]\times\Omega)$, but if a is assumed to be left continuous, a is specified for every $t \in [0,L]$ almost surely.
Remark 3 Even if the system of SFCs lacks its finite elements $(d_{\mathrm{u}}Y, e_i)$, a can be identified.

Proposition 1 (Ogawa integral of H-S integral transform of Wiener functional) Let $f \in \mathcal{L}_1^{1,2}$ and $K \in L^2([0,L]^2)$. Then, $F(t) = \int_0^L K(t,s)f(s)\,ds$ is u-integrable and the Ogawa integral is given by

$$\int_0^L F(t)\,d_{\mathrm{u}}B_t = \int_0^L F(t)\,dB_t + \int_0^L\int_0^L K(t,s)D_t f(s)\,ds\,dt \quad \text{in } \; L^2(\Omega).$$

Theorem 2 (Identification from SFCs-S of stochastic differential) Assume a satisfies the following:

(1) $a(t)$ is real local absolutely continuous a.s.

(2) $a'(t) \in \mathcal{L}_1^{1,2}$, $a(0) \in \mathcal{L}_0^{1,2}$.

(3) $a'(t) \in L^1[0,L]$ a.s. , $\int_0^t D_t a'(s)\,ds \in L^2[0,L]$ a.s.

Then, a and b are identified from the system of SFCs-S $((dX, e_i))_{i\in\mathbb{N}}$ of the stochastic differential $dX_t = a(t)\,dB_t + b(t)\,dt$.

Remark 1 If $a \geq 0 \;\; \lambda \otimes P$-a.e. , then a is identified in the strong sense.
Remark 2 a is specified for every $t \in [0,L]$ almost surely.
Remark 3 Even if the system of SFCs lacks its finite elements (dX, e_i), a can be identified.
Remark 4 If $L < \infty$, then (3) holds.

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Renormalized Gibbs measures and applications to QFT

Fumio Hiroshima

This is a joint work with Oliver Matte in Aalborg university [5]. The Nelson Hamiltonian with UV(ultraviolet) cutoff parameter $\varepsilon > 0$ is given by

$$H_\varepsilon = H_{\rm p} \otimes \mathbb{1} + \mathbb{1} \otimes H_{\rm f} + g\phi(\varrho_\varepsilon(\cdot - x)).$$

Here $H_{\rm p} = -\frac{1}{2}\Delta + V(x)$ is a Schrödinger operator. The operator $H_{\rm f} = d\Gamma(\omega(-i\nabla))$ is the free field Hamiltonian and $\phi(\varrho_\varepsilon(\cdot - x))$ is a Gaussian random variable with cutoff function given by

$$\hat{\varrho}_\varepsilon(k) = \frac{e^{-\varepsilon|k|^2/2}}{\sqrt{\omega(k)}} \mathbb{1}_{|k|>\lambda} \in L^2(\mathbb{R}^3) \quad \varepsilon > 0.$$

E. Nelson [7] introduces the renormalization term:

$$E_\varepsilon = -\frac{1}{2}\int_{|k|>\lambda} \frac{|\hat{\varrho}_\varepsilon(k)|^2}{\omega(k) + |k|^2/2} dk.$$

Here we noice that $E_\varepsilon \to -\infty$ as $\varepsilon \downarrow 0$. It is shown in [8, 7, 1] that there exists a self-adjoint operator $H_{\rm ren}$ such that

$$\lim_{\varepsilon\downarrow 0} e^{-T(H_\varepsilon - g^2 E_\varepsilon)} = e^{-TH_{\rm ren}}.$$

The important fact is that we can *not* see the explicit form of $H_{\rm ren}$, it is however shown in [3] that $H_{\rm ren}$ has the ground state, and it is unique by [6]. Let $\Psi_{\rm g}$ be the ground state of $H_{\rm ren}$. Let $0 \le \phi \in L^2(\mathbb{R}^3)$ and since $(\phi \otimes \mathbb{1}, \Psi_{\rm g}) \neq 0$, we have

$$(\Psi_{\rm g}, O\Psi_{\rm g}) = \lim_{T\to\infty}\lim_{\varepsilon\downarrow 0} \frac{(e^{-TH_\varepsilon}\phi \otimes \mathbb{1}, Oe^{-TH_\varepsilon}\phi \otimes \mathbb{1})}{\|e^{-TH_\varepsilon}\phi \otimes \mathbb{1}\|^2}$$

for any bounded operators O. On the Wiener space $(\Omega, \mathcal{F}, W)$ the finite volume Gibbs measure is defined by

$$\mu_T(A) = \frac{1}{Z_T}\int_{\mathbb{R}^3} dx \mathbb{E}_W^x \left[\mathbb{1}_A \phi(B_{-T})\phi(B_T)e^{\frac{g^2}{2}S_{\rm ren}}\right]$$

for $A \in \mathcal{F}$, where $(B_t)_{t\in\mathbb{R}}$ is BM on the whole line. For some O it follows that $(\Psi_{\rm g}, O\Psi_{\rm g}) = \lim_{T\to\infty} \mathbb{E}_{\mu_T}[O_T]$ with some integrant O_T. Let $\mathcal{F}_{[-S,S]} = \sigma(B_r, r \in [-S, S])$ and we set $\mathcal{G} = \sigma(\cup_{S\ge 0}\mathcal{F}_{[-S,S]})$.

Theorem 0.1 *[4, 2, 5] There exists a probability measure μ_∞ on $(\Omega, \mathcal{G})$ such that $\mu_T \to \mu_\infty$ as $T \to \infty$ in the local weak sense. I.e., $\mu_T(A) \to \mu_\infty(A)$ for $A \in \mathcal{F}_{[-S,S]}$ for arbitrary S.*

We show several applications in terms of infinite volume Gibbs measure μ_∞ in [5].

(1.Super exponential decay of the number of bosons) Let N_Λ be the truncated number operator. Then

$$(\Psi_{\rm g}, e^{+\beta N_\Lambda}\Psi_{\rm g}) = \mathbb{E}_{\mu_\infty}[e^{-(1-e^{+\beta})\int_{-\infty}^0 ds \int_0^\infty dt W_\Lambda}] < \infty$$

for all $\beta \geq 0$.

(2. Gaussian decay) It follows that

$$(\Psi_{\rm g}, e^{+\beta\phi(f)^2}\Psi_{\rm g}) = \frac{1}{\sqrt{1-\beta\|f\|^2/2}}\mathbb{E}_{\mu_\infty}\left[e^{+\frac{\beta S_\infty^2}{2(1-\beta\|f\|^2/2)}}\right].$$

In particular $(\Psi_{\rm g}, e^{\beta\phi(f)^2}\Psi_{\rm g}) < \infty$ for $\beta < 1/(2\|f\|^2)$ and $\lim_{\beta\uparrow 1/(2\|f\|^2)}(\Psi_{\rm g}, e^{\beta\phi(f)^2}\Psi_{\rm g}) = \infty$.

(3. Spatial decay) We have by [6]

$$\Psi_{\rm g} = e^{-T(H_{\rm ren}-E)}\Psi_{\rm g} = e^{TE}e^{-\int_0^T V(B_s+x)ds}e^{\frac{g^2}{2}S_{\rm ren}}e^{a^*(U_T)}e^{-TH_{\rm f}}e^{a(\bar{U}_T)}\Psi_{\rm g}(B_t+x).$$

Here $U_T = -\frac{g}{\sqrt{2}}\int_0^T \frac{e^{-|s|\omega(k)}}{\sqrt{\omega(k)}}e^{-ikB_s}ds$ and $E = \inf\sigma(H_{\rm ren})$. Under some condition on V it follows that

$$\|\Psi_{\rm g}(x)\| \leq Ce^{-c|x|}$$

for a.e.$x \in \mathbb{R}^3$.

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Parisian reflected Lévy processes

Florin AVRAM (University of Pau)
José-Luis PÉREZ (CIMAT)
Kazutoshi YAMAZAKI (Kansai University)[*1]

1. Parisian-reflected Lévy processes

Let $X = (X(t); t \geq 0)$ be a spectrally negative Lévy process defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with its Laplace exponent $\psi(\theta) : [0, \infty) \to \mathbb{R}$, i.e. $\mathbb{E}\big[\mathrm{e}^{\theta X(t)}\big] =: \mathrm{e}^{\psi(\theta)t}$, $t, \theta \geq 0$, given by the *Lévy-Khintchine formula*

$$\psi(\theta) := \gamma\theta + \frac{\sigma^2}{2}\theta^2 + \int_{(-\infty,0)} \big(\mathrm{e}^{\theta x} - 1 - \theta x \mathbf{1}_{\{x>-1\}}\big)\Pi(\mathrm{d}x), \quad \theta \geq 0,$$

where $\gamma \in \mathbb{R}$, $\sigma \geq 0$, and Π is a measure on $(-\infty, 0)$ known as the Lévy measure of X that satisfies $\int_{(-\infty,0)}(1 \wedge x^2)\Pi(\mathrm{d}x) < \infty$. In addition, let $\mathcal{T}_r = \{T(i); i \in \mathbb{N}\}$ be an increasing sequence of epochs of a Poisson process with rate $r > 0$, independent of X.

We construct the *Lévy process with Parisian reflection below* $X_r = (X_r(t); t \geq 0)$ as follows: the process is only observed at times $\mathcal{T}_r$ and is pushed up to 0 if and only if it is below 0. More precisely, we have

$$X_r(t) = X(t), \quad 0 \leq t < T_0^-(1) \tag{1}$$

where

$$T_0^-(1) := \inf\{S \in \mathcal{T}_r : X(S-) < 0\}. \tag{2}$$

The process is then pushed upward by $|X(T_0^-(1))|$ so that $X_r(T_0^-(1)) = 0$. For $T_0^-(1) \leq t < T_0^-(2) := \inf\{S \in \mathcal{T}_r : S > T_0^-(1), X_r(S-) < 0\}$, we have $X_r(t) = X(t) + |X(T_0^-(1))|$. The process can be constructed by repeating this procedure.

Suppose $R_r(t)$ is the cumulative amount of (Parisian) reflection until time $t \geq 0$. Then we have

$$X_r(t) = X(t) + R_r(t), \quad t \geq 0,$$

with

$$R_r(t) := \sum_{i=1}^{\infty} 1_{\{T_0^-(i) \leq t\}} |X_r(T_0^-(i)-)|, \quad t \geq 0, \tag{3}$$

where $(T_0^-(n); n \geq 1)$ can be constructed inductively by (2) and

$$T_0^-(n+1) := \inf\{S \in \mathcal{T}_r : S > T_0^-(n), X_r(S-) < 0\}, \quad n \geq 1.$$

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2. Fluctuation identities

Define

$$\tau_a^-(r) := \inf\{t > 0 : X_r(t) < a\} \quad \text{and} \quad \tau_a^+(r) := \inf\{t > 0 : X_r(t) > a\}, \quad a \in \mathbb{R}.$$

We obtain several fluctuation identities including the following:

1. Joint Laplace transform with killing: for all $q, \theta \geq 0$, $a < 0 < b$, and $x \leq b$,

$$g(x,a,b,\theta) := \mathbb{E}_x\left(e^{-q\tau_b^+(r)-\theta R_r(\tau_b^+(r))}; \tau_b^+(r) < \tau_a^-(r)\right),$$
$$h(x,a,b,\theta) := \mathbb{E}_x\left(e^{-q\tau_a^-(r)-\theta R_r(\tau_a^-(r))}; \tau_a^-(r) < \tau_b^+(r)\right).$$

2. Total discounted values of Parisian reflection: for $a < 0 < b$, $q \geq 0$, and $x \leq b$,

$$f(x,a,b) := \mathbb{E}_x\left(\int_0^{\tau_b^+(r)\wedge\tau_a^-(r)} e^{-qt}\mathrm{d}R_r(t)\right).$$

These can be written in terms of the scale function of the spectrally negative Lévy process.

In the talk, several extensions/modifications of this process are also discussed, including the cases with additional classical reflection from above/below and also the cases X is replaced with a spectrally positive Lévy process.

3. Applications in Insurance

In de Finett's optimal dividend problem, one wants to choose the optimal dividend policy so as to maximize the total expected value of discounted dividends accumulated until ruin.

Consider its version where dividend payments can be made only at the jump times of an independent Poisson process, *a Parisian reflection strategy* is expected to be optimal. Namely, given a suitable barrier b^*, it is optimal to pay dividends at each dividend payment decision time if and only if the surplus is above b^* – the resulting surplus process becomes a Parisian-reflected process. The optimality is shown in [2] for the spectrally negative case with a completely monotone Lévy density, and in [4] for the spectrally positive case.

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A RELATION BETWEEN MODELED DISTRIBUTIONS AND PARACONTROLLED DISTRIBUTIONS

MASATO HOSHINO (WASEDA UNIVERSITY)

In the field of singular SPDEs, there are two big theories: the theory of *regularity structures* [4] by Hairer and the *paracontrolled calculus* [2] by Gubinelli, Imkeller and Perkowski. These two theories are based on a common principle but composed of different mathematical tools. Therefore we can use either of them according to the situation. For example, the former is useful to show a universal property of a large number of SPDEs (e.g. [5, 6]), and the latter is useful to get more detailed information of a specific SPDE (e.g. [3, 7]). However, there is a gap between the two theories about the range of application. For example, the Hairer's theory can be applied to the 3-dimensional parabolic Anderson model

$$(\partial_t - \Delta)u(t,x) = u(t,x)\xi(x), \quad t > 0,\ x \in \mathbb{T}^3,$$

for $\xi \in \mathcal{C}^{-3/2-\epsilon}(\mathbb{T}^3)$ with $\epsilon > 0$, but the GIP theory cannot be.

In this talk, we discuss how to overcome this gap. Recently, Bailleul and Bernicot [1] are tying to improve the GIP theory. Our plan is to complete their work by combining the essence of the Hairer's theory. There is a difference between both theories about the definition of solutions. In the Hairer's theory, the solution is defined as a *modeled distribution*, which represents a local behavior of the solution. In the GIP theory, the solution is defined as a *paracontrolled distribution*, which is defined by nonlocal operators. Each definition has an advantage to each other. We compare these two notions and aim to find a better way.

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The invariant measure and the flow associated to the Φ^4_3-quantum field model

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We consider the invariant measure and flow for the stochastic quantization equation associated to the Φ^4_3-model on the torus, which appears in quantum field theory. By virtue of Hairer's breakthrough, such nonlinear stochastic partial differential equations became solvable and are intensively studied now. In this talk, we present a direct construction to both a global solution and invariant measure for this equation.

Let $m_0 > 0$, Λ be the 3-dimensional torus i.e. $\Lambda = (\mathbb{R}/\mathbb{Z})^3$, and μ_0 be the centered Gaussian measure on the space of Schwartz distributions $\mathcal{S}'(\Lambda)$ with the covariance operator $[2(-\triangle+m_0^2)]^{-1}$. We remark that μ_0 is different from the Nelson's Euclidean free field measure by the scaling $\sqrt{2}$. In order to adjust our setting to those of known results, we define μ_0 as above. In the constructive quantum field theory, there was a problem to construct a measure

$$\mu(d\phi) = Z^{-1} \exp\left(-U(\phi)\right) \mu_0(d\phi)$$

where

$$U(\phi) = \int_\Lambda \left(\frac{1}{4}\phi(x)^4 - C\phi(x)^2 \right) dx$$

and Z is the normalizing constant. Since the support of μ_0 is in the space of tempered distributions, ϕ^4 and ϕ^2 are not defined in usual sense. So, we approximate ϕ and take the limit.

Let $\langle f, g\rangle$ be the inner product on $L^2(\Lambda;\mathbb{C})$. For $k \in \mathbb{Z}^d$, define $e_k(x) := e^{2\pi i k\cdot x}$ where $k\cdot x := k_1x_1 + k_2x_2 + k_3x_3$. For $N \in \mathbb{N}$, denote $\{j \in \mathbb{Z}; |j| \le N\}$ by $\mathbb{Z}_N$, and let P_N be the mapping from $\mathcal{S}'(\Lambda)$ to $L^2(\Lambda;\mathbb{C})$ given by

$$P_N\phi := \sum_{k\in\mathbb{Z}^3_N} \langle \phi, e_k\rangle e_k.$$

Define a function U_N on $\mathcal{S}'(\Lambda)$ by

$$U_N(\phi) = \int_\Lambda \left\{ \frac{1}{4}(P_N\phi)(x)^4 - \frac{3}{2}\left(C_1^{(N)} - 3C_2^{(N)}\right)(P_N\phi)(x)^2 \right\} dx$$

where

$$C_1^{(N)} = \frac{1}{2}\sum_{k\in\mathbb{Z}^3_N} \frac{1}{k^2+m_0^2}, \quad C_2^{(N)} = \frac{1}{2}\sum_{l_1,l_2\in\mathbb{Z}^3_N} \frac{1}{(l_1^2+m_0^2)(l_2^2+m_0^2)(l_1^2+l_2^2+(l_1+l_2)^2+3m_0^2)}.$$

We remark that $\lim_{N\to\infty} C_1^{(N)} = \lim_{N\to\infty} C_2^{(N)} = \infty$, and $C_1^{(N)}$ and $C_2^{(N)}$ are called renormalization constants. Consider the probability measure μ_N on $\mathcal{S}'(\Lambda)$ given by

$$\mu_N(d\phi) = Z_N^{-1} \exp\left(-U_N(\phi)\right) \mu_0(d\phi)$$

where Z_N is the normalizing constant. We remark that $\{\mu_N\}$ is the approximation sequence of the Φ^4_3-measure which will be constructed below as the invariant measure of the associated flow.

Now we consider the stochastic quantization equation associated to $\{\mu_N\}$ as follows.

$$\begin{cases} d\tilde{X}^N_t(x) &= dW_t(x) - (-\triangle + m_0^2)\tilde{X}^N_t(x)dt \\ &\quad -\left\{ P_N[(P_N\tilde{X}^N_t)^3](x) - 3\left(C_1^{(N)} - 3C_2^{(N)}\right) P_N\tilde{X}^N_t(x) \right\} dt \\ \tilde{X}^N_0(x) &= \xi_N(x) \end{cases}$$

where $W_t(x)$ is a white noise with parameter $(t,x) \in [0,\infty) \times \Lambda$ and $\xi_N(x)$ is a random variable which has μ_N as the law, and independent of W_t. We remark that μ_N is the invariant measure with respect to the semigroup generated by the solution to the equation. Let $X^N := P_N\tilde{X}^N$ for $N \in \mathbb{N}$. Then, X^N satisfies the stochastic partial differential equation

$$\begin{cases} dX^N_t(x) &= P_N dW_t(x) - (-\triangle + m_0^2)X^N_t(x)dt \\ &\quad -\left\{ P_N[(X^N_t)^3](x) - 3(C_1^{(N)} - 3C_2^{(N)})X^N_t(x) \right\} dt \\ X^N_0(x) &= P_N\xi_N(x) \end{cases} \tag{1}$$

To apply the Hairer's reconstruction method, which enables us to transform (1) for a solvable partial differential equation, we supplementary introduce the infinite-dimensional Ornstein-Uhlembeck process Z as follows. Let Z be the solution to the stochastic partial differential equation on Λ

$$\begin{cases} dZ_t(x) &= dW_t(x) - (-\triangle + m_0^2)Z_t(x)dt, \quad (t,x) \in [0,\infty) \times \Lambda \\ Z_0(x) &= \zeta(x), \qquad\qquad\qquad\qquad\qquad x \in \Lambda \end{cases}$$

where ζ is a random variable which has μ_0 as its law and is independent of W_t and ξ_N. Let $X^{N,(2)}_t := X^N_t - \mathcal{Z}^{(1,N)}_t + \mathcal{Z}^{(0,3,N)}_t$ for $t \in [0,\infty)$ where

$$\mathcal{Z}^{(0,3,N)}_t := \int_0^t e^{(t-s)(\triangle - m_0^2)} \left(P_N(P_N Z_s)^3 - 3C_1^N P_N Z_s \right) ds, \quad t \in [0,\infty),$$

and decompose $X^{N,(2)}$ into $X^{N,(2),<}$ and $X^{N,(2),\geqslant}$ by means of paraproduct. Then, we have a solvable, coupled, semilinear and dissipative parabolic partial differential equation, which the pair $(X^{N,(2),<}, X^{N,(2),\geqslant})$ satisfies. By applying the technique of the semilinear and dissipative parabolic equation, we obtain some estimates for $X^{N,(2),<}$ and $X^{N,(2),\geqslant}$, which yields the tightness of $X^{N,(2)}$. As the result we obtain the following theorem for the Φ^4_3-measure and the associated flow.

Theorem 1. *For $\varepsilon \in (0,1]$ sufficiently small, $\{X^N\}$ is tight on $C([0,\infty); B^{-1/2-\varepsilon}_{4/3})$, where $B^s_{p,r}$ is the Besov space. Moreover, if X is a limit of a subsequence $\{X^{N(k)}\}$ of $\{X^N\}$ on $C([0,\infty); B^{-1/2-\varepsilon}_{4/3})$, then X is a continuous Markov process on $B^{-1/2-\varepsilon}_{4/3}$, the limit measure μ of the associated subsequence $\{\mu_{N(k)}\}$ is an invariant measure with respect to X and it holds that*

$$\int \|\phi\|^4_{B_4^{-1/2-\varepsilon}} \mu(d\phi) < \infty.$$

Operads and Stochastic Calculus

Roland Friedrich

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In this talk we shall discuss the operadic nature of Itô and Stratonovich calculus. More precisely, we shall be concerned with the space of continuous semimartingales and we will relate it to two fundamental operads which have their roots in algebraic topology. Further, we will explain how the two are related from a cohomological perspective. Finally, we shall present an algebraic-geometric description of the Girsanov transformation.

Supersymmetry in Wiener Space

Jiro Akahori

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I will discuss a probabilistic representation of Sato's grassmanian using stochastic area, which is based on a Fermionic "path-integral"(work by H. Aihara). I will also comment on its links to some other representations and the index theorem.

A FUNCTIONAL CENTRAL LIMIT THEOREM FOR NON-SYMMETRIC RANDOM WALKS ON STEP-r NILPOTENT COVERING GRAPHS

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(Joint work with S. Ishiwata (Yamagata University) and H. Kawabi (Okayama University))

A locally finite, connected and oriented graph $X = (V, E)$ is called a Γ-*nilpotent covering graph* if X is a covering graph of a finite graph $X_0 = (V_0, E_0)$ with a covering transformation group Γ which is a torsion free, finitely generated and nilpotent group of step $r\,(r \geq 2)$. Here V is a set of all vertices and E is a set of all oriented edges in X. For $e \in E$, the origin, the terminus and the inverse edge of e are denoted by $o(e), t(e)$ and $\overline{e}$, respectively. $E_x := \{e \in E \,|\, o(e) = x\}$ denotes the set of all edges whose origin is $x \in V$.

As is known, nilpotent covering graphs can be regarded as an extension of *crystal lattices* or *groups of polynomial growth* and the long time asymptotics for random walks (RWs) on them has been studied by several authors intensively and extensively (for instance, see [4, 1, 2]). We established a functional central limit theorem (FCLT; the Donsker–type invariance principle) for non-symmetric RWs on Γ-nilpotent covering graphs from a viewpoint of *discrete geometric analysis* developed by Sunada [5] under the assumption that Γ is free of step two (see [3]). However, we can completely relax the assumption and obtain an improved result on the FCLT. In this talk, we revisit this problem as a continuation of [3].

Let us consider a Γ-nilpotent covering graph $X = (V, E)$. We introduce a 1-step positive transition probability $p : E \longrightarrow (0, 1]$ which is invariant under Γ-actions. Then the transition probability p induces a RW $\{w_n\}_{n=0}^{\infty}$ with values in X. We may also consider the RW $\{\pi(w_n)\}_{n=0}^{\infty}$ on the quotient graph $X_0 = (V_0, E_0)$ by the Γ-invariance of p. Here $\pi : X \longrightarrow X_0$ is a covering map. Let $m : V_0 \longrightarrow (0, 1]$ be a normalized invariant measure on X_0 and we also write $m : V \longrightarrow (0, 1]$ for the Γ-invariant lift of m to X. Let $\mathrm{H}_1(X_0, \mathbb{R})$ and $\mathrm{H}^1(X_0, \mathbb{R})$ be the first homology group and the first cohomology group of X_0, respectively. In order to measure the homological drift of the RW, we define the *homological direction* of the RW on X_0 by $\gamma_p := \sum_{e \in E_0} p(e)m\big(o(e)\big)e \in \mathrm{H}_1(X_0, \mathbb{R})$. We call the RW on X_0 (m-)*symmetric* if $p(e)m\big(o(e)\big) = p(\overline{e})\big(t(e)\big)$ $(e \in E_0)$. It is clear that the RW is (m-)symmetric if and only if $\gamma_p = 0$.

By the celebrated theorem of Malćev, we find a connected and simply connected nilpotent Lie group G of step r such that Γ is isomorphic to the cocompact lattice in G. By virtue of the general theory of Lie algebras, its Lie algebra $\mathfrak{g}$ may have the direct sum decomposition $\mathfrak{g} = \mathfrak{g}^{(1)} \oplus \mathfrak{g}^{(2)} \oplus \cdots \oplus \mathfrak{g}^{(r)}$ satisfying $[\mathfrak{g}^{(i)}, \mathfrak{g}^{(j)}] \subset \mathfrak{g}^{(i+j)} (i + j \leq r)$ and $\mathfrak{g}^{(i+1)} = [\mathfrak{g}^{(1)}, \mathfrak{g}^{(i)}]\,(i = 1, \dots, r-1)$. Now we take a canonical surjective linear map $\rho_{\mathbb{R}} : \mathrm{H}_1(X_0, \mathbb{R}) \longrightarrow \mathfrak{g}^{(1)}$ through the covering map π. By the discrete analogue of Hodge–Kodaira theorem (cf. [4]), an inner product

$$\langle\!\langle \omega, \eta \rangle\!\rangle_p := \sum_{e \in E_0} p(e)m\big(o(e)\big)\omega(e)\eta(e) - \langle \omega, \gamma_p \rangle \langle \eta, \gamma_p \rangle \quad \big(\omega, \eta \in \mathrm{H}^1(X_0, \mathbb{R})\big)$$

associated with the transition probability p is induced from the space of (modified) harmonic 1-forms on X_0 to $\mathrm{H}^1(X_0, \mathbb{R})$. Using the canonical map $\rho_{\mathbb{R}}$, we construct a flat metric g_0 called the *Albanese metric* on $\mathfrak{g}^{(1)}$ from the inner product $\langle\!\langle \cdot, \cdot \rangle\!\rangle_p$.

We consider a Γ-periodic realization $\Phi_0 : X \longrightarrow G$. In what follows, we take a reference point $x_* \in V$ such that $\Phi_0(x_*) = \mathbf{1}_G$. We call $\Phi_0 : X \longrightarrow G$ *modified harmonic* if

$$\sum_{e \in E_x} p(e) \log\Big(\Phi_0\big(o(e)\big)^{-1} \cdot \Phi_0\big(t(e)\big)\Big)\Big|_{\mathfrak{g}^{(1)}} = \rho_{\mathbb{R}}(\gamma_p) \quad (x \in V).$$

We note that such Φ_0 is uniquely determined, however, the modified harmonic realization has the ambiguity in the components corresponding to $\mathfrak{g}^{(2)} \oplus \cdots \oplus \mathfrak{g}^{(r)}$. More precisely, $\log\big(\Phi_0(x)\big)\big|_{\mathfrak{g}^{(2)} \oplus \cdots \oplus \mathfrak{g}^{(r)}}$ $(x \in V)$ is not be determined uniquely. The quantity $\rho_{\mathbb{R}}(\gamma_p)$ is called the *asymptotic direction*. We note that $\gamma_p = 0$ implies $\rho_{\mathbb{R}}(\gamma_p) = \mathbf{0}_{\mathfrak{g}}$, however, the converse does not hold in general.

We introduce the family of *dilation operators* $\{\tau_\varepsilon\}_{\varepsilon \geq 0}$ acting on G. Let d_{CC} denote the Carnot–Carathéodory metric on G. Note that (G, d_{CC}) is not only a metric space but a geodesic space. Let $\mathcal{D}_n = \{k/n : k = 0, 1, \ldots, n\}$ be a $(1/n$-)partition of the time interval $[0, 1]$ and set $\mathcal{Y}^{(n)}_{k/n} = \tau_{n^{-1/2}} \Phi_0(w_k)\, (n \in \mathbb{N},\, k = 0, 1, \ldots, n)$. Let $(\mathcal{Y}^{(n)}_t)_{0 \leq t \leq 1}$ be the G-valued continuous stochastic process given by the d_{CC}-geodesic interpolation of $\{\mathcal{Y}^{(n)}_{k/n}\}_{k=0}^n$. We write

$$H^1_{\mathbf{1}_G}([0,1], G) = \{h : [0,1] \to G \,|\, h : \text{absolutely continuous}, \, h_0 = \mathbf{1}_G \text{ and } \|\dot{h}\|_{L^2} < \infty\}$$

for the usual Cameron–Martin subspace of the path space, where we are convinced $\dot{h}(t)$ belongs to the evaluation $\mathfrak{g}^{(1)}_{h(t)}$ and $\|\dot{h}\|_{L^2} := \int_0^1 \|\dot{h}(t)\|_{\mathfrak{g}^{(1)}}\, dt$. We denote by $\|\cdot\|_{\alpha\text{-Höl}}$ the α-Hölder norm with respect to d_{CC}. For every small parameter $\varepsilon > 0$, we set

$$\mathcal{W}^{1/2-\varepsilon}_{\mathbf{1}_G}([0,1], G) = \overline{H^1_{\mathbf{1}_G}([0,1], G)}^{\|\cdot\|_{(1/2-\varepsilon)\text{-Höl}}}.$$

Let $(Y_t)_{0 \leq t \leq 1}$ be the G-valued diffusion process which solves the SDE

$$dY_t = \sum_{i=1}^d V_i(Y_t) \circ dB^i_t + \beta(\Phi_0)(Y_t)\, dt, \quad Y_0 = \mathbf{1}_G,$$

where $\{V_1, \ldots, V_d\}$ be an orthonormal basis of $(\mathfrak{g}^{(1)}, g_0)$, the drift coefficient $\beta(\Phi_0) \in \mathfrak{g}^{(2)}$ is defined by

$$\beta(\Phi_0) := \sum_{e \in E_0} p(e) m\big(o(e)\big) \log\Big(\Phi_0\big(o(e)\big)^{-1} \cdot \Phi_0\big(t(e)\big)\Big)\Big|_{\mathfrak{g}^{(2)}},$$

and $(B_t)_{0 \leq t \leq 1} = (B^1_t, \ldots, B^d_t)_{0 \leq t \leq 1}$ is an $\mathbb{R}^d$-valued standard BM.

Then the refinement of the FCLT obtained in [3] is now stated as follows:

Theorem. (1) *Let $\Phi_0, \widetilde{\Phi}_0$ be two modified harmonic realizations. Then $\beta(\Phi_0) = \beta(\widetilde{\Phi}_0)$ holds.*

(2) *Under the assumption that $\rho_{\mathbb{R}}(\gamma_p) = \mathbf{0}_{\mathfrak{g}}$, we have, for every $\varepsilon > 0$,*

$$(\mathcal{Y}^{(n)}_t)_{0 \leq t \leq 1} \Longrightarrow (Y_t)_{0 \leq t \leq 1} \text{ in } \mathcal{W}^{1/2-\varepsilon}_{\mathbf{1}_G}([0,1]; G) \text{ as } n \to \infty.$$

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Free fields and extended Dirichlet spaces

Masatoshi Fukushima

Free fields for Euclidean domains are used to be understood by mathematical physicists as Gaussian random distributions related to the Sobolev spaces of order 1. In 1985, M.Roeckner introduced and studied the Gaussian fields indexted by the general transient extended Dirichlet spaces and those indexed by the signed measures of finite 0-order energy as well. We intend to generalize them to recurrent cases. This talk concerns the Gaussian fields indexed by signed measures of finite logarithmic energy and their relations to Liouville random measures.

Characterization of the explosion time for the Komatu–Loewner evolution

Takuya Murayama (Kyoto University)

1 Introduction

The Komatu–Loewner equation (K–L equation for short) is a correspondence to the Loewner equation in multiply connected domains. Bauer and Friedrich [1] established its concrete expression in standard slit domains of the upper half plane $\mathbb{H}$, and then, Chen, Fukushima et al. [2], [3], [4] investigated some properties of the Komatu–Loewner evolution generated by it.

In this talk, I will give a behavior of the image domain at the exlplosion time of this evolution, which is a refinement of a part of the study in [1]. The proof is based on a probabilistic expression of the solution that was developed in [2], [3] and [4], together with a general theory of complex analysis.

2 Notation and main results

We fix $N \in \mathbb{N}$. Let $C_j \subset \mathbb{H}$, $1 \leq j \leq N$, be slits parallel to the real axis and $K := \bigcup_{j=1}^{N} C_j$. We call a domain of the form $\mathbb{H} \setminus K$ a *standard slit domain*. The left and right end points of the slit C_j are denoted by $z_j = x_j + iy_j$ and $z_j^r = x_j^r + iy_j$ respectively. The slits $\{C_j; 1 \leq j \leq N\}$ are identified with a vector $\mathbf{s} = (y_1, \ldots, y_N, x_1, \ldots, x_N, x_1^r, \ldots, x_N^r) \in \mathbb{R}^{3N}$. We define the set $\mathcal{S}$ of all such "slit vectors" in $\mathbb{R}^{3N}$ as

$$\mathcal{S} := \{\mathbf{s} = (y_1, \ldots, y_N, x_1, \ldots, x_N, x_1^r, \ldots, x_N^r); y_j > 0, \text{ and } x_j < x_k^r \text{ or } x_k < x_j^r \text{ if } y_j = y_k \ (j \neq k)\}.$$

The slits and standard slit domain determined by $\mathbf{s} \in \mathcal{S}$ are denoted by $C_j(\mathbf{s})$, $1 \leq j \leq N$, and $D(\mathbf{s})$, respectively.

Take a standard slit domain $D = D(\mathbf{s})$ and a simple curve $\gamma : [0, t_\gamma) \to \overline{D}$ satisfying $\gamma(0) \in \partial\mathbb{H}$ and $\gamma(0, t_\gamma) \subset D$. For each $t \in [0, t_\gamma)$, there is a unique conformal map g_t from D onto another standard slit domain $D_t = D(\mathbf{s}(t))$ with the *hydrodynamic normalization*

$$g_t(z) = z + \frac{a_t}{z} + o(z^{-1}), \quad z \to \infty,$$

for some $a_t \geq 0$. The image $\xi(t) := g_t(\gamma(t))(= \lim_{z \to \gamma(t)} g_t(z)) \in \partial\mathbb{H}$ of the terminal point $\gamma(t)$ is called the *driving function* of g_t. The quantity a_t, called the *half plane capacity* of the set $\gamma[0, t]$ relative to g_t, is strictly increasing and continuous in t. Thus we can reparametrize the curve γ in such a way that $a_t = 2t$. Under these settings, $g_t(z)$ satisfies the following *K–L equation*:

$$\frac{d}{dt} g_t(z) = -2\pi \Psi_{\mathbf{s}(t)}(g_t(z), \xi(t)), \quad g_0(z) = z \in D. \tag{1}$$

The function $\Psi_{\mathbf{s}}(\cdot, w) = \Psi_D(\cdot, w)$, $w \in \partial\mathbb{H}$, is the conformal map from D onto some standard slit domain with w mapped to ∞, ∞ to 0 and $\Psi_D(z, w) \sim -\pi^{-1}(z - w)^{-1}$ as $z \to w$.

Since $\Psi_{\mathbb{H}}(z, w) = -\pi^{-1}(z - w)^{-1}$, the celebrated Loewner equation

$$\frac{d}{dt} g_t(z) = \frac{2}{g_t(z) - \xi(t)}, \quad g_0(z) = z \in \mathbb{H},$$

corresponds to the K–L equation in $D = \mathbb{H}$.

The end points $z_j(t) = x_j(t) + iy_j(t)$ and $z_j^r(t) = x_j^r(t) + iy_j(t)$ of the slits $C_{j,t} = C_j(\mathbf{s}(t))$ also satisfy the *K–L equation for slits*:

$$\begin{aligned}
\frac{d}{dt} y_j(t) &= -2\pi \Im \Psi_{\mathbf{s}(t)}(z_j(t), \xi(t)), \\
\frac{d}{dt} x_j(t) &= -2\pi \Re \Psi_{\mathbf{s}(t)}(z_j(t), \xi(t)), \\
\frac{d}{dt} x_j^r(t) &= -2\pi \Re \Psi_{\mathbf{s}(t)}(z_j^r(t), \xi(t)).
\end{aligned} \tag{2}$$

We now follow this procedure in the opposite direction. Namely, for a given driving function $\xi \in C([0,\infty);\mathbb{R})$, we first solve the K–L equation (2) for slits $\mathbf{s}(t)$ and then solve (1) for $g_t(z)$, $z \in D$. We denote by t_ξ the explosion time for the ODEs (2) and put $F_t := \{z \in D; t_z \le t\}$, $t < t_\xi$, where

$$t_z = t_\xi \wedge \sup\{t > 0; |g_t(z) - \xi(t)| > 0\}, \quad z \in D,$$

is the explosion time of $g_t(z)$. It is possible to check that g_t, $t \in [0, t_\xi)$, is a unique conformal map from $D \setminus F_t$ onto $D_t = D(\mathbf{s}(t))$ satisfying the hydrodynamic normalization with $a_t = 2t$. The bounded set F_t is not necessarily a curve but a (compact) $\mathbb{H}$-*hull* in the sense that $F_t = \mathbb{H} \cap \overline{F_t}$ and that $\mathbb{H} \setminus F_t$ is simply connected. We call both g_t and F_t the *Komatu–Loewner evolution driven by* $\xi(t)$.

It is a natural problem what happens if t_ξ is finite. A reasonable guess is that the evolution F_t should hit the slits $\bigcup_j C_j$ at the time t_ξ. In terms of the slits $C_{j,t} = C_j(\mathbf{s}(t))$ of D_t, this means that $C_{j,t}$ is absorbed into the real axis for some j as claimed in [1, Theorem 4.1]. Justifying this description is, however, not trivial because the solution to (2) belongs to the space of slits $\mathcal{S}$, not $\mathbb{R}^{3N}$, and the slits $C_{j,t}$ may degenerate to one point or collide with each other before reaching $\partial\mathbb{H}$.

Our main theorem justifies the above description in the following manner:

Theorem 1. *Let $R(w, \mathbf{s}) := \min_{1 \le j \le N} \operatorname{dist}(C_j(\mathbf{s}), w)$ for $w \in \partial\mathbb{H}$ and $\mathbf{s} \in \mathcal{S}$. If $t_\xi < \infty$, then it holds that $\lim_{t \nearrow t_\xi} R(\xi(t), \mathbf{s}(t)) = 0$.*

For the proof, it suffices to extend the solution $\mathbf{s}(t)$ beyond t_ξ if the conclusion does not hold. To this end, we interpret the complicated evolution g_t and F_t in D as a simpler one g_t^0 and F_t in $\mathbb{H}$ by "forgetting the slits," the technique employed in [4]. g_t^0 and F_t extend to a Loewner evolution in $\mathbb{H}$ over the time interval $[0, t_\xi]$. Then, by a version of Carathéodory's kernel theorem (cf. [5, Theorem 15.4.7]), $\{g_t \circ (g_t^0)^{-1}; t < t_\xi\}$ extends to a family of conformal maps over $[0, t_\xi]$. This implies that the limit $\mathbf{s}(t_\xi) = \lim_{t \nearrow t_\xi} \mathbf{s}(t)$ still represents N slits in $\mathbb{H}$, which is a contradiction.

If time permits, I will provide an example where the explosion time for the *stochastic Komatu–Loewner evolution (SKLE)* is finite with probability one. We define the *domain constant* k as

$$k(w, \mathbf{s}) := 2\pi \lim_{z \to w} \left(\Psi_{\mathbf{s}}(z, w) + \frac{1}{\pi} \frac{1}{z - w} \right), \quad w \in \partial\mathbb{H}, \mathbf{s} \in \mathcal{S}.$$

$\mathrm{SKLE}_{\sqrt{6},k}$ is a K–L evolution driven by the random function ξ determined by the system of SDEs (2) and

$$d\xi(t) = -k(\xi(t), \mathbf{s}(t))dt + \sqrt{6}dB_t \tag{3}$$

where B_t is a one-dimensional standard Brownian motion. Then we have the following:

Proposition 2. *Let ζ be the explosion time for the SDEs* (2) *and* (3)*. It holds that $\zeta < \infty$ almost surely.*

This is proven by interpreting SLE_6 as $\mathrm{SKLE}_{\sqrt{6},k}$, i.e., "recalling the slits," which is an idea in the opposite direction to the proof of Theorem 1.

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Distributional Itô's Formula and Regularization of Generalized Wiener Functionals

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Let $X = (X_t)_{t \geqslant 0}$ be a diffusion process defined as a solution to d-dimensional stochastic differential equation

$$\mathrm{d}X_t = \sigma(X_t)\mathrm{d}w(t) + b(X_t)\mathrm{d}t, \quad X_0 = x \in \mathbb{R}^d,$$

where $w = (w^1(t), \cdots, w^d(t))_{t \geqslant 0}$ is a d-dimensional Wiener process with $w(0) = 0$. The main conditions on $\sigma = (\sigma^i_j)_{1 \leqslant i,j \leqslant d}$ and $b = (b^i)_{1 \leqslant i \leqslant d}$ under which we will work are combinations from the following.

(H1) the coefficients σ and b are C^∞, and have bounded derivatives in all orders $\geqslant 1$.

(H2) $(\sigma\sigma^*)(x)$ is strictly positive, where $x = X_0$ and σ^* is the transposed matrix of σ.

(H3) There exists $\lambda > 0$ such that

$$\lambda|\xi|^2_{\mathbb{R}^d} \leqslant \langle \xi, (\sigma\sigma^*)(x)\xi \rangle_{\mathbb{R}^d} \quad \text{for all } \xi \in \mathbb{R}^d,$$

where $\langle \bullet, \bullet \rangle_{\mathbb{R}^d}$ is the standard inner product on $\mathbb{R}^d$, and $|\bullet|_{\mathbb{R}^d} = |\bullet|$ is the corresponding norm.

(H4) There exists $\kappa > 0$ such that

$$\langle \xi, (\sigma\sigma^*)(x)\xi \rangle_{\mathbb{R}^d} \leqslant \kappa|\xi|^2_{\mathbb{R}^d} \quad \text{for all } \xi \in \mathbb{R}^d.$$

In the case of $d = 1$, many researchers in stochastic analysis would know, at least intuitively, the symbol "$\int_0^T \delta_y(X_t)\mathrm{d}t$" stands for a quantity relating to the local time of X at y (evaluated at time T) which is a random variable at each time T, even though not for $\delta_y(X_t)$. Therefore, one expects naturally that the integration with respect to time gives rise to something like a 'smoothing effect'.

One way to formulate this phenomenon might be to employ notions in Malliavin calculus. For a distribution Λ on $\mathbb{R}^d$, if $\Lambda(X_t) \in \mathbb{D}^s_p$ (where $\mathbb{D}^s_p$ stands for the Sobolev space of integrability-index p and differentiability-index $s \in \mathbb{R}$ with respect to the Malliavin derivative), we define $\int_0^T \Lambda(X_t)\mathrm{d}t$ the Bochner integral of the mapping $(0, T] \ni t \mapsto \Lambda(X_t) \in \mathbb{D}^s_p$. Here, one needs to address the Bochner integrability.

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To include local times for one-dimensional diffusions in our scope, we prepare the following

Proposition 1. *Assume* $d = 1$, (H1) *and* (H2). *Let* $\Lambda \in \mathscr{S}'(\mathbb{R})$ *be positive. Then for every* $p \in (1, \infty)$, *we have* $\int_0^T \|\Lambda(X_t)\|_{p,-2} \mathrm{d}t < +\infty$.

Hence the mapping $(0, T] \ni t \mapsto \delta_y(X_t) \in \mathbb{D}_p^{-2}$ is Bochner integrable in the case of $d = 1$. For multi-dimensional cases, it is sufficient to assume $x \neq y$ in order to guarantee the Bochner integrability.

Let $H_p^s(\mathbb{R}^d) := (1 - \triangle)^{-s/2} L_p(\mathbb{R}^d, \mathrm{d}x)$, $p \in (1, \infty)$, $s \in \mathbb{R}$ be the Bessel potential spaces. The main result in this talk is the following.

Theorem 2. *Assume* (H1), (H3) *and* (H4). *Let* $p \in (1, \infty)$ *and* $s \in \mathbb{R}$. *Then for each* $\Lambda \in H_p^s(\mathbb{R}^d)$, *we have*

(i) $\Lambda(X_t) \in \mathbb{D}_{p'}^s$ *for* $t > 0$ *and* $p' \in (1, p)$;

(ii) *if* $p > 2$, *we further have* $\int_{t_0}^T \Lambda(X_t) \mathrm{d}t \in \mathbb{D}_{p'}^{s+1}$ *for* $t_0 \in (0, T]$ *and* $p' \in [2, p)$.

Hence in this sense, we exhibited the 'smoothing effect' of the Bochner integral.

This will be shown by generalizing the classical Itô's formula to our distributional setting. For this, one needs to define the stochastic integrals where the integrand is a family of generalized Wiener functionals. We define this object as (a generalization of) Skorokhod integrals.

Now the key to prove Theorem 2 is to use the elliptic regularity theorem and to track the differentiability-index of the stochastic integrals according to that of the integrand:

Theorem 3. *Assume* (H1) *and* (H2). Let $s \in \mathbb{R}$, $p \geqslant 2$ and $\Lambda \in H_p^s(\mathbb{R}^d)$. Then we have

$$\int_0^T \Lambda(X_t) \mathrm{d}w^i(t) \in \mathbb{D}_p^s, \quad \text{for } i = 1, \cdots, d$$

provided either one of the following

(i) $\lim_{t \downarrow 0} \|\Lambda(X_t)\|_{p,s} = 0$.

(ii) $s \geqslant 0$ and $\int_0^T \|\Lambda(X_t)\|_{p,s}^2 \mathrm{d}t < \infty$.

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Multiray generalization of the arcsine laws for occupation times of infinite ergodic transformations

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1 Introduction

In this talk, we consider a certain distributional convergence of occupation time ratios for ergodic transformations preserving an infinite measure. We give a general limit theorem which can be regarded as a multiray extension of the 2-ray result by Thaler–Zweimüler [4]. Our general limit theorem can be applied to the following problems:

1. (Lamperti's process [2]) Let $Z = (Z_k)_{k\geq 0}$ be an irreducible and null-recurrent discrete-time Markov chain on a countable discrete state space $\{0\}+\sum_{i=1}^{d} A_i$, where $A_1, \dots, A_d$ will be called the *rays*, having the following property: *Z cannot skip the origin 0 when it changes rays*, i.e., the condition $Z_n \in A_i$ and $Z_m \in A_j$ for some $n < m$ and $i \neq j$ implies the existence of $n < k < m$ for which $Z_k = 0$. Then, as $n \to \infty$,

$$\frac{1}{n}\left(\sum_{k=0}^{n-1} \mathbb{1}_{A_1}(Z_k), \dots, \sum_{k=0}^{n-1} \mathbb{1}_{A_d}(Z_k)\right) \to ?$$

2. (interval map with indifferent fixed points [3]) Let $[0,1]$ be decomposed into $[0,1] = \sum_{i=1}^{d} I_i$ for disjoint intervals $I_1, \dots, I_d$, and suppose that the map $T : [0,1] \to [0,1]$ satisfies the following conditions: for each i,

   (a) $T|_{I_i}$ belongs to $C^2(I_i)$ and has a C^2-extension over $\overline{I_i}$, and $\overline{T(I_i)} = [0,1]$,

   (b) there exists $x_i \in I_i$ such that

$$Tx_i = x_i, \quad T'x_i = 1, \quad \text{and}$$
$$(x - x_i)T''x > 0 \ \text{ for any } x \in I_i \setminus \{x_i\}.$$

   In particular, $T' > 1$ on $I_i \setminus \{x_i\}$.

   Let A_i's be disjoint small neighborhoods of x_i's, respectively, and take $Y := [0,1] \setminus \sum_{i=1}^{d} A_i$. We will call $A_1, \dots, A_d$ the *rays* and Y the *origin set*. Note that we can take the rays sufficiently small so that *the orbit $(T^k x)_{k\geq 0}$ cannot skip the origin set when it changes rays*. In this setting, we know that $n^{-1}\sum_{k=0}^{n-1} \mathbb{1}_{\sum_{i=1}^{d} A_i}(T^k x) \to 1$, a.e., as $n \to \infty$. Then, as $n \to \infty$,

$$\frac{1}{n}\left(\sum_{k=0}^{n-1} \mathbb{1}_{A_1}(T^k x), \dots, \sum_{k=0}^{n-1} \mathbb{1}_{A_d}(T^k x)\right) \to ?$$

2 Main results

Let $(X, \mathcal{A}, \mu)$ be a standard measurable space with a σ-finite measure such that $\mu(X) = \infty$, and let $T : (X, \mathcal{A}, \mu) \to (X, \mathcal{A}, \mu)$ be a conservative, ergodic, measure preserving transformation (which is abbreviated by *CEMPT*). Assume that X is decomposed into

$X = Y + \sum_{i=1}^d A_i$ for $Y \in \mathcal{A}$ with $\mu(Y) \in (0,\infty)$ and $A_i \in \mathcal{A}$ with $\mu(A_i) = \infty$ such that the orbit $(T^k x)_{k\ge 0}$ cannot skip the origin set Y when it changes rays $A_1, \dots, A_d$. Set

$$S_n := \left(\sum_{k=0}^{n-1} 1_{A_1} \circ T^k, \dots, \sum_{k=0}^{n-1} 1_{A_d} \circ T^k \right)$$

For $\alpha \in [0,1]$ and $\beta = (\beta_1, \dots, \beta_d) \in [0,1]^d$ with $\sum_{i=1}^d \beta_i = 1$, we write $\zeta_{\alpha,\beta}$ for a $[0,1]^d$-valued random variable whose distribution is characterized as follows:

(1) If $0 < \alpha < 1$, the $\zeta_{\alpha,\beta}$ is equal in distribution to $(\xi_1, \dots, \xi_d)/\sum_{i=1}^d \xi_i$, where $\xi_1, \dots, \xi_d$ are $\mathbb{R}_+$-valued independent random variables with the one-sided α-stable distributions characterized by $\mathbb{E}\left[\exp(-\lambda\xi_i)\right] = \exp(-\beta_i \lambda^\alpha),\ \lambda > 0,\ i = 1, \dots, d.$

(2) If $\alpha = 1$, the $\zeta_{1,\beta}$ is equal a.s. to the constant β.

(3) If $\alpha = 0$, the distribution of $\zeta_{0,\beta}$ is $\sum_{i=1}^d \beta_i \delta_{e^{(i)}}$ with $e^{(i)} = (1_{\{i=j\}})_{j=1}^d \in [0,1]^d$ for $i = 1, \dots, d$.

The $\zeta_{\alpha,\beta}$ are called multidimensional generalized arcsine distributions, and appear as the limits of the joint distribution of the occupation time ratios of diffusions on multiray. See [1] and [5]. We now give our general limit theorem as follows.

Theorem 2.1. *Under certain conditions, the following hold.*

(1) *If S_n/n under $\nu' \xrightarrow{\mathrm{d}} \zeta$ as $n \to \infty$ for some probability measure $\nu' \ll \mu$, then $\zeta \overset{\mathrm{d}}{=} \zeta_{\alpha,\beta}$ for some α and β, and S_n/n under $\nu \xrightarrow{\mathrm{d}} \zeta_{\alpha,\beta}$ as $n \to \infty$ for any probability measure $\nu \ll \mu$.*

(2) *Let $\alpha \in [0,1)$ and $\beta_1, \dots, \beta_d \neq 0$. Then the following are equivalent:*

  (i) *S_n/n under $\nu \xrightarrow{\mathrm{d}} \zeta_{\alpha,\beta}$ as $n \to \infty$ for any probability measure $\nu \ll \mu$.*

  (ii) *There exists a regularly varying function R at ∞ with index $-\alpha$ such that*

$$\mu(x \in Y\ ;\ Tx, \dots, T^n x \in A_i) \sim \beta_i R(n), \ \text{as } n \to \infty,\ i = 1, \dots, d.$$

The case $d = 2$ was due to [4]. The proof in [4] was based on the moment method, which does not seem to be suitable for our multiray case. We adopt instead the double Laplace transform method, which was utilized in the study [1] of occupation times of diffusions on multiray. We will also explain applications to Lamperti's processes and interval maps with indifferent fixed points.

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Controlled Kufarev-Loewner equations and the Sato-Segal-Wilson Grassmannian

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In this talk, which is based on joint work with Takafumi Amaba, we shall be concerned with the lifted Kufarev-Loewner equation to the universal Grassmannian and its relation to the tau function. In order to achieve this, a precise technical understanding of the variation of the involved Faber polynomials and Grunsky coefficients is necessary. An important role is played by the action of the Witt algebra. In order to put the talk(s) into perspective some background information will be given about its origin which is algebro-geometric conformal field theory and integrable systems.

Arbitrage theory in large financial markets

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1 Introduction

In mathematical finance classical market models consist of an $\mathbb{R}^d$-valued semimartingale on some probability space which describes the discounted price process of d financial assets. In this talk we consider a large financial market which consists of infinitely many financial assets. This concept was introduced by Y. Kabanov and D. Kramkov [1] to formalize a market where hundreds of financial assets are available and then several notions and characterizations of arbitrage in large markets were developed [2].

An arbitrage opportunity is the possibility to make a profit in a financial market without risk. The principle of no-arbitrage states that a mathematical model of a financial market should not allow for arbitrage opportunities. The condition of no-arbitrage is essentially equivalent to the existence of an equivalent martingale measure for the price process and this is crucial to the modern theory of finance such as the option pricing theory or the utilty maximization problem.

2 Generalized strategies and arbitrage

We consider a large financial market model consisting of a sequence of semimartingales $\mathbb{S} = \{(S_t^n)_{t\in[0,T]}\}_{n\in\mathbb{N}}$ on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,T]}, \mathbb{P})$ which describes the discounted price process of infinitely many financial assets. We denote the n-th small market by $\mathbb{S}^n = (S^k)_{k\leq n}$.

Definition 1.

- *A strategy in the n-th small market is an $\mathbb{R}^n$-valued predictable process which is $\mathbb{S}^n$-integrable. The wealth process corresponding to a strategy H is the stochastic integral $H \bullet \mathbb{S}^n$.*
- *We say that a strategy H in the n-th small market is admissible if its wealth process $H \bullet \mathbb{S}^n$ is bounded from below.*
- *We denote the set of all admissible strategies in the n-th small market by $\mathcal{H}^n$ and the set of attainable claims in the n-th small market by $\mathcal{K}^n = \{(H \bullet \mathbb{S}^n)_T | H \in \mathcal{H}^n\}$.*
- *We say that the n-th small market satisfies NA (No Arbitrage) if $\mathcal{K}^n \cap L^0_+ = \{0\}$ where L^0_+ denotes the convex cone of nonnegative random variables on $(\Omega, \mathcal{F}, \mathbb{P})$.*

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The admissibility condition is imposed to exclude so called "doubling strategies". The condition NA says that there exists no admissible strategy which satisfies $(H \bullet \mathbb{S}^n)_T \geq 0$ *a.s.* (without risk) and $\mathbb{P}\{(H \bullet \mathbb{S}^n)_T > 0\} > 0$ (positive profit).

The corresponding notions of trading strategies or arbitrage in a large financial market $\mathbb{S}$ are as follows;

Definition 2.

- *For each $n \in \mathbb{N}$, let H^n be an $\mathbb{R}^n$-valued, predictable, $\mathbb{S}^n$-integrable process. A sequence $\mathbb{H} = (H^n)_{n\in\mathbb{N}}$ is called generalized strategy if $(H^n \bullet \mathbb{S}^n)$ converges in the Emery topology to a semimartingale Z, which is called a generalized stochastic integral (or a generalized wealth process) and denoted by $Z = \mathbb{H} \bullet \mathbb{S}$.*
- *A generalized strategy $\mathbb{H} = (H^n)_{n\in\mathbb{N}}$ is called admissible if the approximating sequence $(H^n)_{n\in\mathbb{N}}$ is uniformly admissible.*
- *We denote the set of all generalized admissible strategies by $\mathcal{H}$ and the set of approximately attainable claims in the large market by $\mathcal{K} = \{(\mathbb{H} \bullet \mathbb{S})_T | \mathbb{H} \in \mathcal{H}\}$.*
- *We say that a large market satisfies NGA (No Generalized Arbitrage) if $\mathcal{K} \cap L^0_+ = \{0\}$.*

The notion of generalized stochastic integral with respect to a sequence of semimartingales was introduced by De. Donno and Pratelli [3], which formalizes the idea of a trading strategy in which each asset can contribute, possibly with an infinitesimal weight.

3 A change of numéraire

We deal with the change of numéraire problem in large financial markets. Consider a model $\mathbb{X} = ((S^n)_{n\in\mathbb{N}}, 1, V)$, where V is a positive semimartingale describing a new numéraire (that is, a new currency unit). If the currency unit is changed to the new numéraire V, the price process $\mathbb{X}$ will be multiplied by the exchange ratio $\frac{1}{V}$ and the price process under this new numéraire becomes $\mathbb{Z} = ((\frac{S^n}{V})_{n\in\mathbb{N}}, \frac{1}{V}, 1)$. We will talk about the condition under which the NGA condition is preserved under a change of numéraire.

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Discrete approximations for non-colliding SDEs

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joint work with
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Abstract

In this talk, we consider discrete approximations for non-colliding particle systems. We introduce a semi-implicit Euler-Maruyama approximation which preservers the non-colliding property for some class of non-colliding particle systems and provide a strong rate of convergence in L^p-norm. We also consider some modified explicit/implicit Euler-Maruyama schemes.

Non-colliding particle systems

A non-colliding particle systems $X = (X(t) = (X_1(t), \dots, X_d(t)))_{t \geq 0}$ is a solution of the following system of stochastic differential equations (SDEs)

$$\mathrm{d}X_i(t) = \left\{ \sum_{j \neq i} \frac{\gamma_{i,j}}{X_i(t) - X_j(t)} + b_i(X_i(s)) \right\} dt + \sum_{j=1}^{d} \sigma_{i,j}(X(t)) \mathrm{d}W_j(t), \quad i = 1, \dots, d, \qquad (1)$$

with $X(0) \in \Delta_d = \{\mathbf{x} = (x_1, \dots, x_d)^* \in \mathbb{R}^d : x_1 < x_2 < \cdots < x_d\}$, $\gamma_{i,j} = \gamma_{j,i} \geq 0$ and $W = (W(t) = (W_1(t), \dots, W_d(t))^*)_{t \geq 0}$ a d-dimensional standard Brownian motion.

The existence and uniqueness of a strong non-colliding solution to (1) have been studied intensively by many others. However, there are still few results on the numerical approximation for such kind of systems. To the best of our knowledge, the paper of Li and Menon [4] is the only work in this direction. Li and Menon introduced an explicit "tamed" Euler-Maruyama approximation since the coefficient b_i are super linear growth. However, their scheme unfortunately does not preserve the non-colliding property of a solution, which is an important characteristic of the SDE (1).

Recently, many authors study numerical approximation for one-dimensional SDEs with boundary (e.g. Bessel process $\mathrm{d}X_t = dt/X_t + \mathrm{d}W_t, X_t > 0$ and CIR process $\mathrm{d}X_t = (a - bX_t)dt + X_t^{1/2}\mathrm{d}W_t, X_t > 0$). Dereich, Neuenkirch and Szpruch [2] introduced an implicit Euler-Maruyama scheme for CIR process and showed that the rate of convergence is $1/2$, and extended to one-dimensional SDEs with boundary condition by Alfonsi [1] and Neuenkirch and Szpruch [5].

Discrete approximations for non-colliding particle systems

Inspired by [1, 2, 5], we define a semi-implicit Euler-Maruyama scheme for a solution of non-colliding SDE (1) as follows: $X^{(n)}(0) := X(0)$ and for each $k = 0, \dots, n-1$, $X^{(n)}(t_{k+1}^{(n)})$ is defined as the unique solution in Δ_d of the following equation:

$$X_i^{(n)}(t_{k+1}^{(n)}) = X_i^{(n)}(t_k^{(n)}) + \left\{ \sum_{j\neq i} \frac{\gamma_{i,j}}{X_i^{(n)}(t_{k+1}^{(n)}) - X_j^{(n)}(t_{k+1}^{(n)})} + b_i\left(X_i^{(n)}(t_k^{(n)})\right) \right\} \frac{T}{n}$$
$$+ \sum_{j=1}^{d} \sigma_{i,j}\left(X^{(n)}(t_k^{(n)})\right)\left\{W_j(t_{k+1}^{(n)}) - W_j(t_k^{(n)})\right\},$$

where $t_k^{(n)} := kT/n$. Since the equation

$$\xi_i = a_i + \sum_{j\neq i} \frac{c_{i,j}}{\xi_i - \xi_j}, \quad i = 1, \ldots, d,$$

has a unique solution in Δ_d for each $a_i \in \mathbb{R}$ and $c_{i,j} \geq 0$ with $c_{i,i+1} > 0$ (see Proposition 2.2 in [6]), thus $X^{(n)} = (X^{(n)}(t_k^{(n)}))_{k=0,\ldots,n}$ is well-defined for each $n \in \mathbb{N}$.

In this talk, under some assumptions on the constants $\gamma_{i,j}$ and the coefficients b_i, $\sigma_{i,j}$, we will show that the SDE (1) has a unique global strong solution on Δ_d and the Euler-Maruyama approximation $X^{(n)}$ converges to the unique solution to the non-colliding SDE (1) in L^p-sense for some $p \geq 1$ or 2 with convergence rate $n^{1/2}$ or n. More preciously, we will show that there exists $C > 0$ such that,

$$\mathbb{E}\left[\sup_{k=1,\ldots,n} |X(t_k^{(n)}) - X^{(n)}(t_k^{(n)})|^p\right]^{1/p} \leq \begin{cases} Cn^{-1/2}, & \text{if } b_i \text{ are Lipschitz continuos and } p \geq 1, \\ Cn^{-1}, & \text{if } b_i \in C_b^2(\mathbb{R};\mathbb{R}) \text{ and } p \geq 2, \end{cases}$$

(see Theorem 2.8 and 2.9 in [6]). Note that the singular coefficients $\frac{1}{x_i - x_j}$ make the system difficult to deal with. In order to overcome this obstacle, we need an upper bound for both moments and inverse moments of $X_i(t) - X_j(t)$.

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Heat trace asymptotics for equiregular sub-Riemannian manifolds

Yuzuru Inahama *

This is a jointwork with Setsuo Taniguchi (Kyushu University) and can be found at arXiv Preprint Server (arXiv:1706.02450).

We study a "div-grad type" sub-Laplacian with respect to a smooth measure and its associated heat semigroup on a compact equiregular sub-Riemannian manifold. We prove a short time asymptotic expansion of the heat trace up to any order. Our main result holds true for any smooth measure on the manifold, but it has a spectral geometric meaning when Popp's measure is considered. Our proof is probabilistic. In particular, we use S. Watanabe's distributional Malliavin calculus.

In Introduction of his textbook on sub-Riemannian geometry [3], R. Montgomery emphasized the importance of spectral geometric problems in sub-Riemannian geometry by asking "Can you 'hear' the sub-Riemannian metric from the spectrum of its sublaplacian?" (Of course, this is a slight modification of M. Kac's renowned question.) In the same paragraph, he also mentioned Malliavin calculus, which is a powerful infinite-dimensional functional analytic method for studying stochastic differential equations (SDEs) under the Hörmander condition on the coefficient vector fields.

However, there is no canonical choice of measure on a general sub-Riemannian manifold and hence no canonical choice of sub-Laplacian. Therefore, in order to pose spectral geometric questions, one should consider a subclass of sub-Riemannian manifolds. In this regard, the class of equiregular sub-Riemannian manifolds seems suitable for the following reason. As Montgomery himself proved in Section 10.6, [3], there exists a canonical smooth volume called Popp's measure on an equiregular sub-Riemannian manifold. Popp's measure is determined by the sub-Riemannian metric only.

In this talk we prove a short time asymptotic expansion of the heat trace up to an arbitrary order on a compact equiregular sub-Riemannian manifold. Our main tool is Watanabe's distributional Malliavin calculus.

Let $M = (M, \mathcal{D}, g)$ be a sub-Riemannian manifold and μ be a smooth volume on M. ($\mathcal{D}$ is a subbundle of TM that satisfies the Hörmander condiyion at every point and g is an inner product on $\mathcal{D}$.) We study the second-order differential operator of the form $\triangle = \mathrm{div}_\mu \nabla^{\mathcal{D}}$, where $\nabla^{\mathcal{D}}$ is the horizontal gradient in the direction of $\mathcal{D}$ and div_μ is the divergence with respect to μ. (In our convention, $\triangle$ is a non-positive operator.) By the way it is defined, $\triangle$ with its domain being $C_0^\infty(M)$ is clearly symmetric on $L^2(\mu)$. If M is compact, then $\triangle$ is known to be essentially self-adjoint on $C^\infty(M)$ and $e^{t\triangle/2}$ is of trace class for every $t > 0$, where $(e^{t\triangle/2})_{t\geq 0}$ is the heat semigroup associated with $\triangle/2$.

Now we are in a position to state our main result in this paper. As we have already mentioned, it has a spectral geometric meaning when μ is Popp's measure.

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Theorem 1 *Let M be a compact equiregular sub-Riemannian manifold of Hausdorff dimension ν and let μ be a smooth volume on M. Then, we have the following asymptotic expansion of the heat trace:*

$$\mathrm{Trace}(e^{t\triangle/2}) \sim \frac{1}{t^{\nu/2}}(c_0 + c_1 t + c_2 t^2 + \cdots) \qquad \textit{as } t \searrow 0 \tag{1}$$

for certain constants $c_0 > 0$ and $c_1, c_2, \ldots \in \mathbb{R}$.

Since the asymptotic expansion in Theorem 1 is up to an arbitrary order, we can prove meromorphic prolongation of the spectral zeta function associated with $\triangle$ by a standard argument. Denote by $0 = \lambda_0 < \lambda_1 \leq \lambda_2 \leq \cdots$ be all the eigenvalues of $-\triangle$ in increasing order with the multiplicities being counted and set

$$\zeta_\triangle(s) = \sum_{i=0}^{\infty} \lambda_i^{-s} \qquad (s \in \mathbb{C},\, \Re s > \frac{\nu}{2}).$$

By the Tauberian theorem, the series on the right hand side absolutely converges and defines a holomorphic function on $\{s \in \mathbb{C} \mid \Re s > \nu/2\}$.

Corollary 2 *Let assumptions be the same as in Theorem 1. Then, $\zeta_\triangle$ admits a meromorphic prolongation to the whole complex plane $\mathbb{C}$.*

To the best of our knowledge, Theorem 1 and Corollary 2 seem new for a general compact equiregular sub-Riemannian manifold. It should be noted, however, that the leading term of the asymptotics (1) is already known. See Métivier (1976) for example. No explicit value of c_0 is known in general. For some concrete examples or relatively small classes of compact equiregular sub-Riemannian manifolds, the full asymptotic expansion (1) or the meromorphic extension of the spectral zeta function was proved. Most of such classes are subclasses of step-two or corank-one sub-Riemannian manifolds.

Our proof of Theorem 1 is based on Takanobu's beautiful result [2] on the short time asymptotic expansion of hypoelliptic heat kernels on $\mathbb{R}^d$ on the diagonal. Using results in Taniguchi (1983) and Grong-Thalmaier (2016)/Thalmaier (2016), we can do the same thing on a compact manifold. (The former developed manifold-valued Malliavin calculus under the partial Hörmander condition, while the latter constructed $\triangle/2$-diffusion process on M via stochastic parallel transport.)

References

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