Existence and uniqueness of strict solutions of stochastic linear evolution equations in M-type 2 Banach spaces

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1. Introduction

We study non-autonomous abstract stochastic evolution equations of the form

$$\begin{cases} dX + A(t)X\,dt = F(t)dt + G(t)dW(t), & 0 < t \le T, \\ X(0) = \xi, \end{cases} \tag{1.1}$$

in a complex separable Banach space $(E, \|\cdot\|)$ of M-type 2. Here, $\{A(t), t \ge 0\}$ is a family of densely defined, closed linear operators in E; $W(t)$ is a cylindrical Wiener process on separable Hilbert space U and is defined on a filtered probability space $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathbb{P})$; F is an E-valued predictable function; G is an $L_2(U;E)$-valued predictable function where $L_2(U;E)$ denotes the space of Hilbert-Schmidt operators; and ξ is an $\mathcal{F}_0$-measurable random variable. We suppose that A, F and G satisfy the following structural assumptions.

(A1) For all $t \in [0,T]$, the spectrum $\sigma(A(t))$ and the resolvent of $A(t)$ satisfy

$$\sigma(A(t)) \subset \Sigma_\varpi = \{\lambda \in \mathbb{C} : |\arg \lambda| < \varpi\}$$

and

$$\|(\lambda - A(t))^{-1}\| \le \frac{M_\varpi}{|\lambda|}, \qquad \lambda \notin \Sigma_\varpi$$

with some constants $\varpi \in (0, \frac{\pi}{2})$ and $M_\varpi > 0$.

(A2) There exists an exponent $\nu \in (0,1]$ such that

$$\mathcal{D}(A(s)) \subset \mathcal{D}(A(t)^\nu), \qquad 0 \le s,t \le T.$$

(A3) There exist an exponent $\mu \in (1-\nu, 1]$ and a constant $N > 0$ such that

$$\|A(t)^\nu [A(t)^{-1} - A(s)^{-1}]\| \le N|t-s|^\mu, \qquad 0 \le s,t \le T.$$

(F1) There exist $\beta \in (0,1]$ and $0 < \sigma < \min\{\beta, \mu+\nu-1\}$ such that $F \in \mathcal{F}^{\beta,\sigma}((0,T];E)$ a.s., where $\mathcal{F}^{\beta,\sigma}((0,T];E)$ denotes the weighted Hölder continuous function space.

(G1) There exist a constant $\delta > \frac{1}{2}$ and a square-integrable random variable ζ such that

$$\|A(t)^\delta G(t) - A(s)^\delta G(s)\|_{L_2(U;E)} \le \zeta |t-s|^\sigma \qquad \text{a.s.}$$

and $\mathbb{E}\|A(0)^\delta G(0)\|^2_{L_2(U;E)} < \infty$.

2. Main results

Theorem 2.1 (Uniqueness [1]). *Let* (A1), (A2), (A3) *be satisfied. If there exists a strict solution to the equation* (1.1) *then it is unique.*

Theorem 2.2 (Existence [1]). *Let* (A1), (A2), (A3), (F1) *and* (G1) *be satisfied. Suppose that* $\xi \in \mathcal{D}(A(0)^\beta)$ *a.s. and* $\mathbb{E}\|A(0)^\beta \xi\|^2 < \infty$. *Then there exists a unique strict solution of* (1.1) *possessing the regularity:*

$$A^\beta X \in \mathcal{C}([0,T];E), \quad X \in \mathcal{C}^{\gamma_1}([0,T];E) \qquad a.s.$$

and

$$AX \in \mathcal{C}^{\gamma_2}([\epsilon,T];E) \qquad a.s.$$

for every $0 < \gamma_1 < \min\{\beta, \frac{1}{2}\}$, $0 < \gamma_2 < \min\{\delta - \frac{1}{2}, \sigma\}$ *and* $\epsilon \in (0,T]$. *In addition,* X *satisfies the estimate*

$$\begin{aligned}\mathbb{E}\|A^\beta X(t)\|^2 \leq & C[\mathbb{E}\|A(0)^\beta \xi\|^2 + \mathbb{E}\|F\|^2_{\mathcal{F}^{\beta,\sigma}} \\ & + \mathbb{E}\|A(0)^\delta G(0)\|^2_{L_2(U;E)} t^{1-2(\beta-\delta)} + t^{1-2(\beta-\delta)+2\sigma}]\end{aligned}$$

for the case $\beta \geq \delta$ *and*

$$\mathbb{E}\|A^\beta X(t)\|^2 \leq C[\mathbb{E}\|A(0)^\beta \xi\|^2 + \mathbb{E}\|F\|^2_{\mathcal{F}^{\beta,\sigma}} + \mathbb{E}\|A(0)^\delta G(0)\|^2_{L_2(U;E)} t + t^{1+2\sigma}]$$

for the case $\beta < \delta$. *Furthermore, if* $A(0)^\delta G(0) = 0$ *then*

$$X \in \mathcal{C}^{\gamma_1}([0,T];E) \qquad \text{for every } 0 < \gamma_1 < \min\{\frac{1+\sigma}{2}, \delta, \beta\}.$$

In the favorable case $\nu = 1$ (see (A2)), the condition (G1) in Theorem 2.2 can be replaced by a simplified one, say

(G1)' There exist a constant $\delta_1 > \dfrac{1}{2}$ and a square-integrable random variable $\bar{\zeta}$ such that

$$\|A(0)^{\delta_1}[G(t) - G(s)]\|_{L_2(U;E)} \leq \bar{\zeta}|t-s|^\sigma \qquad \text{a.s.}$$

and $\mathbb{E}\|A(0)^{\delta_1} G(t)\|^2_{L_2(U;E)} < \infty$ for every $t \in [0,T]$.

Theorem 2.3 ([1]). *If* (G1)' *takes place then so does* (G1).

References

[1] T. V. Ton, Y. Yamamoto, A. Yagi, Existence and uniqueness of strict solutions of stochastic linear evolution equations in M-type 2 Banach spaces, 24 pages, arXiv:1508.07431v2.