L^p Solutions of Backward Stochastic Differential Equations and their Malliavin Derivatives

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Backward Stochastic Differential Equations

Let $(\Omega, \mathcal{F}, P)$ be a complete probability space, $W = (W_t)_{0 \le t \le T}$ be an n-dimensional standard Brownian motion defined on the space, and $(\mathcal{F}_t)_{0 \le t \le T}$ be the Browinian filtration augmented by all P-null sets. $T > 0$ represents a terminal time.

We consider the following d dimensional stochastic differential equation:

$$-dY_t = f(t, Y_t, Z_t)dt - Z_t^* dW_t, \quad Y_T = \xi,$$

which is often rewritten in the form:

$$Y_t = \xi + \int_t^T f(s, Y_s, Z_s)ds - \int_t^T Z_s^* dW_s, \quad 0 \le t \le T,$$

where ξ is an $\mathcal{F}_T$-measurable random variable which represents a terminal condition, $f\colon [0,T] \times \Omega \times \mathbb{R}^d \times \mathbb{R}^{n \times d} \to \mathbb{R}^d$ with progressive measurablity for each element in $\mathbb{R}^d \times \mathbb{R}^{n \times d}$. These type of SDEs are called backward SDEs (BSDEs for short).

For $p > 1$, an L^p solution of a BSDE is a pair (Y, Z), composed of $(\mathcal{F}_t)$-adapted continuous process and $(\mathcal{F}_t)$-progressively measurable process respectively, which satisfies the BSDE and

$$E\left[\sup_{0 \le t \le T} |Y_t|^p\right] + E\left[\left(\int_0^T |Z_t|^2 dt\right)^{\frac{p}{2}}\right] < \infty.$$

L^p solutions and their Malliavin derivatives

Under appropriate assumptions on ξ and f, it is known that BSDE with respect to ξ and f has unique L^p solution.

Then El Karoui, Peng and Quenez [1] showed the L^p ($p \ge 2$) solution of BSDE is differentiable in Malliavin's sense. In addition, an important property between Y and Z is given; $Z_t = D_t Y_t$, where $D_u = \frac{d}{dt}\big|_{t=u} \nabla$ and ∇ represents the Malliavin derivative operator.

Malliavin derivatives of Wiener functionals take values on Hilbert spaces. They are specifically Hilbert-Schmidt operators. Therefore it is useful to consider BSDEs on Hilbert spaces when trying to differentiate many times. In this talk, we deal with BSDEs on Hilbert spaces and introduce some results on higher order Malliavin differentiability of solutions.

References

[1] N. El Karoui, S. Peng, and M.C. Quenez, Backward stochastic differential equations in finance, *Math. Finance*, 1997, **7**, 1-71.