

# APPROXIMATING ROUGH STOCHASTIC PDES

JAN MAAS

We study a class of vector-valued equations of Burgers type driven by a multiplicative space-time white noise. These equations are of the form

$$(1) \quad \partial_t u = \nu \partial_x^2 u + F(u) + G(u) \partial_x u + \theta(u) \xi ,$$

where the function  $u = u(t, x; \omega) \in \mathbb{R}^n$  is vector-valued. We assume that the functions  $F: \mathbb{R}^n \rightarrow \mathbb{R}^n$  and  $G, \theta: \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$  are smooth and the products in the terms  $G(u) \partial_x u$  as well as in  $\theta(u) \xi$  are to be interpreted as matrix vector multiplication. The noise term  $\xi$  denotes an  $\mathbb{R}^n$ -valued space-time white noise and the multiplication should be interpreted in the sense of Itô integration against an  $L^2$ -cylindrical Wiener process.

In the case where  $G$  is the gradient of a function  $\mathcal{G}$  the equation (1) is classically well-posed. The definition of weak solutions and their construction uses the conservation law structure of (1): The nonlinearity is rewritten as

$$G(u) \partial_x u = \partial_x \mathcal{G}(u),$$

and the derivative can be treated by integration by parts. However, several seemingly natural approximation schemes fail to produce solutions of (1), but converge to different limit equations in which extra terms may appear.

In the case where  $G$  is not a total derivative it is not even clear how to make sense of (1). The solution does not have the regularity required to make sense of the nonlinearity. We use rough path theory to resolve this issue. Weak solutions can be defined by testing against a smooth test function  $\varphi$  and defining the term

$$\int_{-\pi}^{\pi} \varphi(x) G(u(t, x)) \partial_x u(t, x) dx$$

as a rough integral.

We study approximations to (1) of the form

$$du_\varepsilon = \left( \nu \Delta_\varepsilon u_\varepsilon + F(u_\varepsilon) + G(u_\varepsilon) D_\varepsilon u_\varepsilon \right) dt + \theta(u_\varepsilon) H_\varepsilon dW,$$

for a large class of regularisations  $\Delta_\varepsilon$ ,  $D_\varepsilon$ , and  $H_\varepsilon$ . We show that the  $u_\varepsilon$  converge to a process  $\bar{u}$  that solves an equation similar to (1) with an extra term

$$-\Lambda \theta(u) \nabla G(u) \theta^T(u).$$

This term is the local spatial cross variation of  $u$  and  $G(u)$  and can be interpreted as a spatial Itô-Stratonovich correction. The constant  $\Lambda$  depends on the specific choice of the approximations and can be calculated explicitly. We obtain a rate of convergence of  $\varepsilon^{1/6}$ .

This is joint work with Martin Hairer and Hendrik Weber.

## REFERENCES

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