Historical Superprocess Related to Random Measure

ランダム測度に付随するヒストリカル超過程

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1. Superprocess with Branching Rate Functional

We introduce the superprocess with branching rate functional, which forms a general class of measure-valued branching Markov processes with diffusion as a underlying spatial motion. We write as $\langle \mu, f \rangle = \int f d\mu$. For simplicity, $M_F = M_F(\mathbb{R}^d)$ is the space of finite measures on $\mathbb{R}^d$. Define a second order elliptic differential operator $L = \frac{1}{2}\nabla \cdot a\nabla + b \cdot \nabla$, and $a = (a_{ij})$ is positive definite and we assume that $a_{ij}, b_i \in C^{1,\varepsilon} = C^{1,\varepsilon}(\mathbb{R}^d)$. Here $C^{1,\varepsilon}$ indicates the totality of all Hölder continuous functions with index ε $(0 < \varepsilon \leqslant 1)$, allowing their first order derivatives to be locally Hölder continuous. $\{\xi, \Pi_{s,a}\}$ indicates a corresponding L-diffusion. Moreover CAF stands for continuous additive functional. Let $\mathbb{K}^q$ (with $q > 0$) denote the Dynkin class of locally admissible CAF's with index q. When we write C_b as the set of bounded continuous functions on $\mathbb{R}^d$, then C_b^+ is the set of positive members in C_b. The process $\{X, \mathbb{P}_{s,\mu}\}$ is said to be a *superprocess with branching rate functional* K or simply (L, K, μ)-*superprocess* if X is a continous M_F-valued time-inhomogeneous Markov process with Laplace functional $\mathbb{P}_{s,\mu} e^{-\langle X_t, \varphi\rangle} = e^{-\langle \mu, v(s,t)\rangle}$ for $0 \leqslant s \leqslant t$, $\mu \in M_F$ and $\varphi \in C_b^+$. Here v is uniquely determined by the log-Laplace equation

$$\Pi_{s,a}\varphi(\xi_t) = v(s,a) + \Pi_{s,a}\int_s^t v^2(r,\xi_r)K(dr), \qquad 0 \leqslant s \leqslant t, \quad a \in \mathbb{R}^d. \qquad (1)$$

2. Associated Historical Superprocess

The historical superprocess was initially studied by Dynkin (1991) (cf. Dawson-Perkins, 1991). $\mathbb{C}$ denotes the space of continuous paths on $\mathbb{R}^d$ with topology of uniform convergence. To each $w \in \mathbb{C}$ and $t > 0$, we write $w^t \in \mathbb{C}$ as the stopped path of w. We denote by $\mathbb{C}^t$ the totality of all these paths stopped at t. To every $w \in \mathbb{C}$ we associate the corresponding stopped path trajectory $\tilde{w}$ defined by $\tilde{w}_t = w^t$ $(t \geq 0)$. The image of L-diffusion w under the map : $w \mapsto \tilde{w}$ is called the *L-diffusion path process*. Moreover, we define $\mathbb{C}_R^\times = \{(s,w) : s \in \mathbb{R}_+, w \in \mathbb{C}^s\}$ and we denote by $M(\mathbb{C}_R^\times)$ the set of measures η on $\mathbb{C}_R^\times$ which are finite, if restricted to a finite time interval. Let K be a positive CAF of ξ. $\{\tilde{X}, \tilde{\mathbb{P}}_{s,\mu}\}$ is said to be a Dynkin's *historical superprocess* if $\tilde{X}$ is a time-inhomogeneous Markov process with state $\tilde{X}_t \in M_F(\mathbb{C}^t)$, $t \geq s$, with Laplace functional $\tilde{\mathbb{P}}_{s,\mu} e^{-\langle \tilde{X}_t, \varphi\rangle} = e^{-\langle \mu, v(s,t)\rangle}$

for $0 \leqslant s \leqslant t$, $\mu \in M_F(\mathbb{C}^s)$ and $\varphi \in C_b^+(\mathbb{C})$, where v is uniquely determined by the log-Laplace equation

$$\tilde{\Pi}_{s,w_s}\varphi(\tilde{\xi}_t) = v(s,w_s) + \tilde{\Pi}_{s,w_s}\int_s^t v^2(r,\tilde{\xi}_r)\tilde{K}(dr), \quad 0 \leqslant s \leqslant t, \quad w_s \in \mathbb{C}^s. \tag{2}$$

We call this $\tilde{X}$ an associted historical superprocess in Dynkin's sense.

3. Superprocess Related to Random Measure

Suppose that $p > d$, and let $\phi_p(x)$ be the reference function. C denotes the space of continuous functions on $\mathbb{R}^d$, and define $C_p = \{f \in C : |f| \leqslant C_f \cdot \phi_p, \exists C_f > 0\}$. We denote by M_p the set of non-negative measures μ on $\mathbb{R}^d$, satisfying $\langle \mu, \phi_p \rangle < \infty$. It is called the space of p-tempered measures. We define the continuous additive functional K_η of ξ by $K_\eta = \langle \eta, \delta_x(\xi_r)\rangle dr$ for $\eta \in M_p$. Then $X^\eta = \{X_t^\eta; t \geq 0\}$ is said to be a measure-valued diffusion with branching rate functional K_η if for $\mu \in M_F$, X satisfies the Laplace functional $\mathbb{P}_{s,\mu}^\eta e^{-\langle X_t^\eta, \varphi\rangle} = e^{-\langle \mu, v(s)\rangle}$ for $\varphi \in C_b^+$, where the function v is uniquely determined by

$$\Pi_{s,a}\varphi(\xi_t) = v(s,a) + \Pi_{s,a}\int_s^t v^2(r,\xi_r)K_\eta(dr), \quad (0 < s \leqslant t, a \in \mathbb{R}^d). \tag{3}$$

Assume that $d = 1$ and $0 < \nu < 1$. Let $\lambda \equiv \lambda(dx)$ be the Lebesgue measure on $\mathbb{R}$, and let $(\gamma, \mathbb{P})$ be the stable random measure on $\mathbb{R}$ with Laplace functional

$$\mathbb{P}e^{-\langle \gamma, \varphi\rangle} = \exp\left\{-\int \varphi^\nu(x)\lambda(dx)\right\}, \quad \varphi \in C_b^+. \tag{4}$$

Let $p > \nu^{-1}$ in what follows. We consider a positive CAF $K_{\gamma(\omega)}$ of ξ for $\mathbb{P}$-a.a. ω. So that, thanks to Dynkin's general formalism, there exists an (L, K_γ, μ)-superprocess X^γ when we adopt a p-tempered measure γ for K_η instead of η, as far as K_γ may lie in $\mathbb{K}^q$ $(\exists q > 0)$.

4. Historical Superprocess Related to Random Measure

As for the historical superprocess associated with the superprocess X^γ related to random measure, we can prove the following.

THEOREM. (Main Result) *Let K_γ be a positive CAF of ξ lying in the Dynkin class $\mathbb{K}^q$. Then there exists a historical superprocess $\tilde{\mathbb{X}}^\gamma = \{\tilde{X}^\gamma, \tilde{\mathbb{P}}_{s,\mu}^\gamma, s \geq 0, \mu \in M_F(\mathbb{C}^s)\}$ in the Dynkin sense. In fact, $\tilde{\mathbb{X}}^\gamma$ is a time-inhomogeneous Markov process with state $\tilde{X}_t^\gamma \in M_F(\mathbb{C}^t)$, $t \geq s$, with Laplace functional $\tilde{\mathbb{P}}_{s,\mu}^\gamma \exp\{-\langle \tilde{X}_t^\gamma, \varphi\rangle\} = e^{-\langle \mu, v(s,t)\rangle}$ for $0 \leqslant s \leqslant t$, $\mu \in M_F(\mathbb{C}^s)$ and $\varphi \in C_b^+(\mathbb{C})$, where v is uniquely determined by the log-Laplace equation*

$$\tilde{\Pi}_{s,w_s}\varphi(\tilde{\xi}_t) = v(s,w_s) + \tilde{\Pi}_{s,w_s}\int_s^t v^2(r,\tilde{\xi}_r)\tilde{K}_\gamma(\omega; dr), \quad 0 \leqslant s \leqslant t, \quad w_s \in \mathbb{C}^s. \tag{5}$$