Asymptotic expansion theorem of Watanabe for Wiener-Poisson variables.

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The asymptotic expansion theorem for Wiener functionals was obtained by S. Watanabe [1]. Using this, he studied the short time behavior of the fundamental solution to a heat equation. N. Yoshida[2], N, Kunitomo and A. Takahashi[3] have applied this theorem to mathematical finance.

In this talk, we shall discuss the asymptotic expansion theorem on the Wiener-Poisson space. As an application, we shall consider SDE with jumps

$$S_t^{(\epsilon)} = x_0 + \int_0^t b_0\, S_{r-}^{(\epsilon)}\, dr + \epsilon \int_0^t a(\, S_{r-}^{(\epsilon)}\,) \diamond dZ_r,$$

where $\epsilon \in (0,1)$ and Z_t is a Lévy process, and give the following asymptotic expansion formula

$$\mathbf{E}[(K^{(\epsilon)} - S_{T_0}^{(\epsilon)})_+] \sim c_1\epsilon + c_2\epsilon^2 + \cdots,$$

where $K^{(\epsilon)} = e^{bT_0} - \epsilon k_0$ $(k_0 > 0)$.

References

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[1] This is joint work with Yasushi Ishikawa (Ehime university)