ON BEHAVIORS OF MEASURE-VALUED MARKOV PROCESSES WITH IRREGULAR PARAMETERS

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Let D be a domain of $\mathbb{R}^d$. Let $C_c^+(D)$ be the space of non-negative continuous functions on D with compact support. We denote by $C^{k,\eta}(D)$ the usual Hölder space with index $\eta \in (0,1]$, which includes derivatives of order k, and in particular we simply wrire $C^\eta(D)$ instead of $C^{0,\eta}(D)$. Let L be an elliptic operator on the domain D of the form

$$\begin{aligned} L &= \frac{1}{2}\nabla \cdot a\nabla + b \cdot \nabla \\ &= \frac{1}{2}\sum_{i,j=1}^{d} \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial}{\partial x_j}\right) + \sum_{i=1}^{d} b_i(x)\frac{\partial}{\partial x_i}, \end{aligned} \tag{1}$$

where the matrix $a(x) = (a_{ij}(x))$ is symmetric and positive definite for $x \in D$. Suppose that $a_{ij}(x) \in C^{1,\eta}(D)$ and $b_i(x) \in C^{1,\eta}(D)$ for $i, j = 1, 2, \ldots, d$. We often write

$$\langle \nu, f\rangle = \int_D f(x)\nu(\mathrm{d}x) \tag{2}$$

for the integral of measurable function f with respect to the measure ν on D.

We set $\mathcal{L}_0 = L + \beta$ on the domain D. Let λ_c be the generalized principal eigenvalue for $\mathcal{L}_0$.

Theorem 1. *Let $\lambda_c > 0$. Suppose that the operator $\mathcal{L} = L + \beta - \lambda_c$ is subcritical. Let $X = (X_t, \mathbb{P}_\mu)$ be the (L, β, α, D) superprocess. Then we* *have*

$$\lim_{t\to\infty} e^{-\lambda_c t}\mathbb{E}_\mu[\langle X_t, g\rangle] = 0 \tag{3}$$

for any g in the space $C_c^+(D)$.

Next we consider the behaviors of a class of measure-valued Markov processes with irregular parameters. Suppose that $d = 1$ for simplicity.

Let $X = (X_t, \mathbb{P}_\mu)$ be the (L, δ_0, α) superprocess. We denote by λ the generalized principal eigenvalue for the operator $\mathcal{L} = L + \delta_0$, and we define $\mathcal{C} = \{u > 0;\ (\mathcal{L} - \lambda)u = 0\}$. Suppose that $\mathcal{C} \neq \emptyset$.

Theorem 2. *Suppose that $\mathcal{L} - \lambda$ is critical. For $\varphi \in \mathcal{C}$, we have*

$$\lim_{t\to\infty} e^{-\lambda t}\mathbb{E}_{\delta_x}\langle X_t, g\rangle = K \cdot \varphi(x), \tag{4}$$

for any $g \in C_c(\mathbb{R})$ with $K = (\varphi, g)_{L^2}/\|\varphi\|_{L^2}^2$.