An Ito formula for a generalized Bessel process and Skorohod type equation for multivariate Bessel processes

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In this talk, we consider a generalized Bessel process and give an Ito formula for this process, and then apply this formula to characterlize a class of multivariate Bessel procsses as a solution of Skorohod type equation. This work is motivated by our study on approximating queueing network by multivariate Bessel processes. Let $B(t)$ be an adapted Brownian motion with $< B > (t) = \sigma_B^2 t$ and $D(t)$ be an adapted 0-quadratic variation process and we consider the following stochasic differetial equation (SDE):

$$Z(t) = Z(0) + \sigma_B^2 \delta t + 2\int_0^t \sqrt{Z(s)} dB(s)) + 2\int_0^t \sqrt{Z(s)} dD(s)) \quad Z(0) = z$$

where δ is a positive constant. Typical example of $D(t)$ we treat is p-variation processes with $1 \leq p < 2$. When the process $D(t)$ vanishes in the above equation, the process $X(t) = \sqrt{Z(t)}$ is nothing but a Bessel process with dimension δ. Then, for a function g which is twice continuously differentiable except the boudary point 0, we give an Ito formula for the processes $Z(t)$ and $X(t) = \sqrt{Z(t)}$. This formula gives a decomposition of the process $g(Z(t))$ as a Dirichlet process . As an aplication of the formula, we show that a class of multivariate Bessel processes can be obtained as the solution of Skorohod type equation.

References

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