Stochastic flows of SDEs with non-Lipschtzian coefficients driven by multi-dimensional symmetric α stable processes

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Abstract

The construction of flows in the case of Brownian motion was investigated in the begining of the 80's (See [4] for example). In the case of Lévy processes, stochastic flows were studied in depth by [2] and [1], etc. They considered especially the diffeomorphism of stochastic flows where coefficients are sufficiently smooth. In this part, we focus on the construction of stochastic flows under **non-Lipschitz conditions** of the coefficients.

This presentation is organized as follows. In the first section, we discuss non-contact problems of solutions where the Riesz potential operator plays an essential role. In the second section, we summarize the results of the pathwise uniqueness property. Pathwise uniquness guarantees the well-definedness of the mapping from initial data to the solution, $y \mapsto Y_t(y)$. In the third section, we show the continuity of the map with respect to initial data. Here, hypergeometric functions and Bessel functions are the key to solving the problem. The fourth section is devoted to the behavior of the mapping at infinity. Finally, in the last section, combining these properties and applying Jordan's curve theorem, we construct stochastic flows.

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