Conditioning Quadratic Wiener functionals and Plücker coordinates — with a new example

Setsuo Taniguchi (Kyushu Univ.)

Let $T > 0$ and $(\mathcal{W}, \mu)$ be the d-dimensional classical Wiener space over $[0, T]$, i.e., $\mathcal{W}$ is the Banach space of all $\mathbb{R}^d$-valued continuous functions defined on $[0, T]$ starting at the origin, and μ is the Wiener measure on $\mathcal{W}$. Denote by H the Cameron-Martin subspace of $\mathcal{W}$. Thinking of a symmetric Hilbert Schmidt operator $A : H \to H$ as a constant Wiener functional with values in the Hilbert space of Hilbert-Schmidt operators of H to itself, we define the quadratic Wiener functional associated with A by $Q_A = (\nabla^*)^2 A$, where ∇^* stands for the adjoint operator of the Malliavin gradient ∇. For linearly independent $\eta_1, \ldots, \eta_M \in H$ and $N \leq M$, set $\boldsymbol{\eta}^{(N)} = (\nabla^* \eta_1, \ldots, \nabla^* \eta_N)$. Investigated in this talk is the conditioned stochastic oscillatory integral

$$I_N(\zeta) = \int_{\mathcal{W}} e^{\zeta Q_A/2} \delta_0(\boldsymbol{\eta}^{(N)})\, d\mu, \quad \zeta \in \mathbb{C}$$

where δ_0 is the Dirac measure concentrated at $0 \in \mathbb{R}^N$ and $\delta_0(\boldsymbol{\eta}^{(N)})$ denotes the pull-back due to S. Watanabe. The exact expressions of the above integrals $I_N(\zeta)$, $1 \leq N \leq M$, will be given in terms of the Plücker coordinate of a point in the $(M, 2M)$-Grassmannian. Such correspondence was firstly pointed out and investigated by Hara-Ikeda [1] in the case of the classical and generalized Lévy areas. In this talk, we extend their observation to general cases with the help of the Jacobi field approach to quadratic Wiener functional introduced by Ikeda-Manabe [2].

We shall testify our generalization in a new quadratic Wiener functional which is obtained as the Malliavin derivative of the square norm of Brownian sample path. The Wiener functional attracts us since it determines the stationary point of the square norm. Some more detailed observations on the Wiener functional will be presented in the talk.

The talk is based on two recent papers [3,4].

Bibliography

[1] K. Hara and N. Ikeda, Quadratic Wiener functionals and dynamics on Grassmannians, Bull. Sci. math. **125** (2001), 481–528. [2] N. Ikeda and S. Manabe, Van Vleck-Pauli formula for Wiener integrals and Jacobi fields, in "Itô's stochastic calculus and probability theory", Ed. by N. Ikeda, S. Watanabe, M. Fukushima, and H. Kunita, pp.141–156, Springer-Verlag, 1996. [3] S. Taniguchi, On the quadratic Wiener functional associated with the Malliavin derivative of the square norm of Brownian sample path on interval, Electro. Comm. Probab., **11** (2006), 1–10. [4] S. Taniguchi, On the Jacobi field approach to stochastic oscillatory integrals with quadratic phase function, to appear in Kyushu J. Math., MHF Preprint Series 2006-3, available at `http://www.math.kyushu-u.ac.jp/coe/report/mhf.cgi`