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Abstract For any strongly continuous resolvent $\{G_\alpha; \alpha > 0\}$ on a real Hilbert space H, we shall introduce a closed symmetric form $(\mathcal{E}(V), \mathcal{D}(\mathcal{E}(V))$ in a wide sense, to be called G-form, associated with the potential operator $V \equiv \lim_{\alpha \to 0} G_\alpha$ defined by Yosida transformation in Hunt's potential theory, besides the usual closed symmetric form $(\mathcal{E}(A), \mathcal{D}(\mathcal{E}(A))$, to be called D-form in this talk, associated with the infinitesimal generator A.

First, we shall characterize the G-form as a dual form of the D-form and the D-form as a dual form of the G-form through Legendre-Fenchel transformation in convex analysis.

Next, we shall find some relationships between the D-space $\mathcal{D}(\mathcal{E}(A))$ and the G-space $\mathcal{D}(\mathcal{E}(V))$, to be a domain of the D-form and the G-form, respectively. In particular, we shall obtain a fundamental relation between the resolvent $\{G_\alpha; \alpha > 0\}$ associated with the D-form and the resolvent $\{G^\bullet_\alpha; \alpha > 0\}$ associated with the G-form. Furthermore, we shall show that there exists a unitary operator $V^\bullet$ from the completed G-space onto the completed D-space, and characterise its inverse operator and its restriction to the space $\mathcal{D}(\mathcal{E}(V))$. Note that the restriction of the operator $V^\bullet$ to the space $\mathcal{D}(V)$ is the potential operator V. Moreover, we shall construct four kinds of resolvents by extending the resolvents $\{G_\alpha|_{\mathcal{D}(\mathcal{E}(A))}; \alpha > 0\}, \{G^\bullet_\alpha|_{\mathcal{D}(\mathcal{E}(A))}; \alpha > 0\}$ on the completed D-space and $\{G_\alpha|_{\mathcal{D}(\mathcal{E}(V))}; \alpha > 0\}, \{G^\bullet_\alpha|_{\mathcal{D}(\mathcal{E}(V))}; \alpha > 0\}$ on the completed G-space, respectively and then characterize them.

Finally, we shall consider the extended D-space and the extended G-space for a general case and investigate the problem concerning the equivalence of the non-degeneracy of seminorms and the completeness of them.