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確率解析とその周辺

場所：京都大学大学院理学研究科 1 号館 516 号室

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Uniqueness of Gibbs measures on $C(\mathbb{R} \to \mathbb{R})$*

Ichiro SHIGEKAWA† (Kyoto University)

We consider the uniqueness problem of Gibbs measures on $C(\mathbb{R} \to \mathbb{R})$. Suppose we are given a potential function $V : \mathbb{R} \mapsto \mathbb{R}$. We assume that V is continuous and non-negative. In this talk, a Gibbs measure associated with V is formally expressed as

$$\nu(dx) = Z^{-1} \exp\left\{ -\frac{1}{2} \int_{-\infty}^{\infty} |\dot{x}(t)|^2 \, dt - \int_{-\infty}^{\infty} V(x(t)) \, dt \right\} \prod_{t \in \mathbb{R}} dx(t). \tag{1}$$

Precise characterization is fomulated through Dobrushin-Lanford-Ruelle equation as follows. For $I \subseteq \mathbb{R}$, we set $\mathcal{F}_I = \sigma\{x(t); t \in I\}$. Let $P_{s,x}^{t,y}$ be the pinned Brownian motion with $x(s) = x$ and $x(t) = y$. Then a probability measure μ is called a Gibbs measure if it satisfies

$$\mu(\,\cdot\,|\mathcal{F}_{[s,t]^c})(x(\cdot)) = Z^{-1} \exp\left\{ - \int_s^t V(x(u)) du \right\} P_{s,x}^{t,y} \otimes \delta_{x_{[s,t]^c}}. \tag{2}$$

Here Z is a normlizing constant. In this talk, we only deal with Gibbs measures satisfying the tightness condition: we set

$$\mathcal{G} = \{\mu \text{ satisfies DLR equation (1) and the family } \{\mu \circ x(t)^{-1}\} \text{ is tight}\}. \tag{3}$$

This type of measures, or more general classes, were discussed by many orthors, e.g., [1, 2]. We are interested in the uniqueness of $\mathcal{G}$. This measure is closely related to an operator $H = \frac{1}{2}\triangle - V$. H is a self-adjoint operator in $L^2(\mathbb{R})$ and we deonote the spectrum of $-H$ by $\sigma(-H)$. Now define

$$\lambda_0 = \inf \sigma(-H). \tag{4}$$

λ_0 is called a principal eigen-value in general. It is not always an eigenvalue but we can always find a positive solution ϕ such that $-H\phi = \lambda_0 \phi$. If $\phi \in L^2(\mathbb{R})$, then λ_0 is an eigenvalue. Our main theorem is the following:

Theorem 1. If λ_0 is an eigenvalue then $\sharp(\mathcal{G}) = 1$, i.e., the uniqueness holds.

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Large deviation for stochastic line integrals as L^p-currents

Kazumasa Kuwada[*†]

Department of Mathematics, Ochanomizu university

In this talk, we consider the large deviation principle for stochastic line integrals of Brownian paths on a compact Riemannian manifold. We regard them as a random map on a Sobolev space of vector fields. We show that the differentiability order of the Sobolev space can be chosen to be almost independent of the dimension of the underlying space under stronger integrability condition than L^2. The large deviation is formulated for the joint distribution of stochastic line integrals and the empirical distribution of a Brownian path. As the result, the rate function is given explicitly.

Let M be a d-dimensional closed Riemannian manifold with the normalized Riemannian measure m. Assume $d \geq 3$. Fix a constant $L > 0$ satisfying that the Ricci curvature is bounded below by $-(d-1)L$. Let Δ be the Laplace-Beltrami operator and $\Box$ the Hodge-Kodaira Laplacian acting on differential 1-forms. For $p \in [1, \infty]$, let L^p, $\mathscr{A}^{0,p}$ and $\mathscr{X}^{0,p}$ be the Lebesgue space of scalar functions, that of 1-forms and that of vector fields respectively(with respect to m, otherwise stated). For $r > 0$, we define (r,p)-Sobolev spaces by

- $W^{r,p} := (1-\Delta)^{-r/2} L^p$,
- $\mathscr{A}^{r,p} := (L + 1 - \Box)^{-r/2} \mathscr{A}^{0,p}$.

For $f \in W^{r,p}$, $\|f\|_{r,p} := \|(1-\Delta)^{r/2} f\|_{L^p}$. In the same way, we define the Sobolev norm $\|\cdot\|_{r,p}$ (use the same symbol) on $\mathscr{A}^{r,p}$. For $p \in (1,\infty)$, let $\mathscr{X}^{-r,p} = \left(\mathscr{A}^{r,p'}\right)^*$ be the Sobolev space of vector fields of negative differentiability orders for $1/p + 1/p' = 1$. Let us introduce the following condition on a pair of indices (r,p):

(I) $p \in (1,2)$ and $r > 2 + d - d/p$.

Let $(\{X_t\}_{t\geq 0}, \{\mathbb{P}_x\}_{x\in M})$ be the Brownian motion, or the Markov process generated by $\Delta/2$. For $\alpha \in \mathscr{A}$, we can define the stochastic line integral $\int_{X[0,t]} \alpha$ along $\{X_s\}_{s\in[0,t]}$. We regard the stochastic line integral as a Random map $R_t : \mathscr{A} \to \mathbb{R}$ given by $R_t(\alpha) = \int_{X[0,t]} \alpha$. Let $\mathscr{C}^{-r,p} := C([0,\infty) \to \mathscr{X}^{-r,p})$ with the compact uniform convergence topology.

Theorem 1 [1] *For each pair (r,p) satisfying (I), R is realized as a $\mathscr{C}^{-r,p}$-valued random variable by taking a suitable version.*

We denote the space of the probability measures by $\mathscr{M}_+^1$ and the space of the signed measures on M of finite total variation by $\mathscr{M}$. We consider the weak topology on $\mathscr{M}_+^1$ and $\mathscr{M}$. Let $R_t^\lambda := \lambda^{-1} R_{\lambda t}$ and $L_t^\lambda := \lambda^{-1} \int_0^{\lambda t} \delta_{X_s} ds$. Then $(R_1^\lambda, L_1^\lambda)$ is an $\mathscr{X}^{-r,p} \times \mathscr{M}_+^1$-valued random variable.

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[*]joint work with S. Kusuoka(Tokyo) and Y. Tamura(Keio)

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Definition 1 *For* $(\zeta, \mu) \in \mathscr{X}^{-r,p} \times \mathscr{M}$*, we say* $(\zeta, \mu) \in \mathscr{G}$ *if and only if it satisfies the following:*

(i) $\mu \in \mathscr{M}^1_+$*,* $\mu \ll m$ *and* $h_\mu := \sqrt{d\mu/dm} \in W^{1,2}$*.*

(ii) $\operatorname{div} \zeta = 0$ *in the sense of distribution, i.e.,* ${}_{\mathscr{X}^{-r,p}}\langle \zeta, du \rangle_{\mathscr{A}^{r,p'}} = 0$ *for each* $u \in C^\infty(M)$*.*

(iii) *Let* $\bar{\zeta}$ *be given by*

$$\bar{\zeta}(x) = \begin{cases} \dfrac{1}{h_\mu(x)^2}\zeta(x) & \text{if } h_\mu(x) > 0, \\ 0 & \text{otherwise.} \end{cases}$$

Then $\bar{\zeta} \in \mathscr{X}^{0,2}(d\mu)$*.*

We define a rate function I *on* $\mathscr{X}^{-r,p} \times \mathscr{M}$ *by*

$$I(\zeta, \mu) = \begin{cases} \dfrac{1}{2}\displaystyle\int_{\{h_\mu > 0\}} \frac{|\zeta|^2}{h_\mu^2} dm + \frac{1}{2}\int_M |\nabla h_\mu|^2 \, dm & \text{if } (\zeta, \mu) \in \mathscr{G}, \\ \infty & \text{otherwise.} \end{cases}$$

Note that ζ *is regarded as a measurable vector field.*

Theorem 2 [1] *For each pair* (r, p) *satisfying (I),* $\{(R^\lambda_1, L^\lambda_1)\}_{\lambda > 0}$ *satisfies the large deviation principle in* $\mathscr{X}^{-r,p} \times \mathscr{M}^1_+$ *as* $\lambda \to \infty$ *with the convex good rate function* I*. That is, for each* $E \in \mathscr{X}^{-r,p} \times \mathscr{M}^1_+$*,*

$$\limsup_{t\to\infty} \frac{1}{t} \log \left(\sup_{x \in M} \mathbb{P}_x \left[(R^\lambda_1, L^\lambda_1) \in E \right] \right) \le - \inf_{(\zeta,\mu) \in \bar{E}} I(\zeta, \mu),$$
$$\liminf_{t\to\infty} \frac{1}{t} \log \left(\inf_{x \in M} \mathbb{P}_x \left[(R^\lambda_1, L^\lambda_1) \in E \right] \right) \ge - \inf_{(\zeta,\mu) \in E^\circ} I(\zeta, \mu),$$

where $\bar{E}$ *is the closure of* E *and* E° *the interior of* E*.*

In [2], the large deviation principle in the case of $p = 2$ is studied in a bit different formulation. We use this result for proving Theorem 2 via (inverse) contraction principle. The key estimate for both of Theorem 1 and Theorem 2 is the following exponential integrability:

Proposition 1 *Let* $G_r(\zeta) := \sup_{\|\alpha\|_{r,\infty} \le 1} |\zeta(\alpha)|$*. Then, for* $r > 2$*, there exist constants* $\gamma > 0$ *and* $C > 0$ *so that for each* $\eta \in (0, 1]$*,*

$$\sup_{x \in M} \mathbb{E}_x \left[\exp \left(\gamma \eta^{-1/2} \sup_{0 \le t \le \eta} G_r(R_t) \right) \right] < C.$$

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An Ito formula for a generalized Bessel process and Skorohod type equation for multivariate Bessel processes

Keigo YAMADA (Kanagawa University)

In this talk, we consider a generalized Bessel process and give an Ito formula for this process, and then apply this formula to characterlize a class of multivariate Bessel procsses as a solution of Skorohod type equation. This work is motivated by our study on approximating queueing network by multivariate Bessel processes. Let $B(t)$ be an adapted Brownian motion with $< B > (t) = \sigma_B^2 t$ and $D(t)$ be an adapted 0-quadratic variation process and we consider the following stochasic differetial equation (SDE):

$$Z(t) = Z(0) + \sigma_B^2 \delta t + 2\int_0^t \sqrt{Z(s)} dB(s)) + 2\int_0^t \sqrt{Z(s)} dD(s)) \quad Z(0) = z$$

where δ is a positive constant. Typical example of $D(t)$ we treat is p-variation processes with $1 \leq p < 2$. When the process $D(t)$ vanishes in the above equation, the process $X(t) = \sqrt{Z(t)}$ is nothing but a Bessel process with dimension δ. Then, for a function g which is twice continuously differentiable except the boudary point 0, we give an Ito formula for the processes $Z(t)$ and $X(t) = \sqrt{Z(t)}$. This formula gives a decomposition of the process $g(Z(t))$ as a Dirichlet process . As an aplication of the formula, we show that a class of multivariate Bessel processes can be obtained as the solution of Skorohod type equation.

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Stochastic flows of SDEs with non-Lipschtzian coefficients driven by multi-dimensional symmetric α stable processes

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Abstract

The construction of flows in the case of Brownian motion was investigated in the begining of the 80's (See [4] for example). In the case of Lévy processes, stochastic flows were studied in depth by [2] and [1], etc. They considered especially the diffeomorphism of stochastic flows where coefficients are sufficiently smooth. In this part, we focus on the construction of stochastic flows under **non-Lipschitz conditions** of the coefficients.

This presentation is organized as follows. In the first section, we discuss non-contact problems of solutions where the Riesz potential operator plays an essential role. In the second section, we summarize the results of the pathwise uniqueness property. Pathwise uniquness guarantees the well-definedness of the mapping from initial data to the solution, $y \mapsto Y_t(y)$. In the third section, we show the continuity of the map with respect to initial data. Here, hypergeometric functions and Bessel functions are the key to solving the problem. The fourth section is devoted to the behavior of the mapping at infinity. Finally, in the last section, combining these properties and applying Jordan's curve theorem, we construct stochastic flows.

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Asymptotic expansions for the Laplace approximations for Itô functionals of Brownian rough paths

Yuzuru Inahama (Tokyo Institute of Technology) Hiroshi Kawabi (Kyushu University)

Let (X, H, μ) be an abstract Wiener space, Y be a real separable Banach space, $w = (w_t)_{0\le t\le 1}$ be an X-valued Brownian motion. Let also $X^\varepsilon := (X^\varepsilon_t)_{0\le t\le 1}$ $(\varepsilon > 0)$ be a solution of the following formal (Stratonovich) SDE:

$$dX^\varepsilon_t = \sigma(X^\varepsilon_t) \circ \varepsilon dw_t + \sum_{i=1}^N a_i(\varepsilon) b_i(X^\varepsilon_t) dt \quad \text{with } X^\varepsilon_0 = 0. \tag{1}$$

Here, $\sigma \in C_b^\infty(Y, L(X,Y)), b_i \in C_b^\infty(Y,Y), i = 1, \dots, N$, and $a = (a_1, \dots, a_N) : [0,1] \to \mathbb{R}^N$ is a smooth curve. In this talk, we will discuss the Laplace type asymptotic expansion of the functional integral of the form $\mathbb{E}\big[G(X^\varepsilon)\exp(-F(X^\varepsilon/\varepsilon^2))\big]$ as $\varepsilon \searrow 0$. The large deviation was done in Inahama-Kawabi [J. London Math. Soc., 2006]. The Laplace method for the leading term $(= \alpha_0)$ was done in Inahama [JFA, 2006]、Inahama-Kawabi [Proceedings of the Abel Symposium 2005, to appear].

In order to give a precise definition for the Wiener functional X^ε, we introduce some notations. For a real separable Banach B, we set $P(B) := \{x \in C([0,1], B) \mid x_0 = 0\}$, $BV(B) := \{\gamma \in P(B) \mid \|\gamma\|_1 < \infty\}$, and $G\Omega_p(B)$ $(2 < p < 3)$, (which is called the space of geometric rough paths over B). The law of εw on $P(X)$ is denoted by $\mathbb{P}'_\varepsilon$. $\mathcal{H}$ is the Cameron-Martin space for $(P(X), \mathcal{H}, \mathbb{P}'_1)$. In this talk we assume $(|\cdot|_{X\otimes X}, \mu)$ satisfies the exactness condition **(EX)** (cf. Ledoux-Lyons-Qian), which implies the existence of Brownian rough paths $\overline{w} = (1, \overline{w}_1, \overline{w}_2) \in G\Omega_p(X)$. The law of scaled Brownian rough paths $\overline{\varepsilon w}$ on $G\Omega_p(X)$ is denotedd by $\mathbb{P}_\varepsilon$.

Set $\tilde{X} := X \oplus \mathbb{R}^N$ and define $\tilde{\sigma} \in C_b^\infty(Y, L(\tilde{X}, Y))$ by

$$\tilde{\sigma}(y)\big[(x,u)\big]_{\tilde{X}} := \sigma(y)x + \sum_{i=1}^N b_i(y)u_i, \quad y \in Y, x \in X, u = (u_1, \dots, u_N) \in \mathbb{R}^N.$$

We consider the following ODE in the rough path sense:

$$dy_t = \tilde{\sigma}(y_t) d\tilde{x}_t \quad \text{with } y_0 = 0. \tag{2}$$

For an input $\overline{\tilde{x}} \in G\Omega_p(\tilde{X})$, there is a unique solution $\overline{z} = (\overline{\tilde{x}}, \overline{y}) \in G\Omega_p(\tilde{X} \oplus Y)$ in the rough path sense. We denote it by $\overline{y} = \Phi(\overline{\tilde{x}})$ and also call it a solution of ODE (2). The Itô map $\Phi : G\Omega_p(\tilde{X}) \to G\Omega_p(Y)$ is locally Lipschitz continuous. For $\lambda = (\lambda^{(1)}, \dots, \lambda^{(N)}) \in \mathrm{BV}(\mathbb{R}^N), \overline{x} \in G\Omega_p(X)$, define $\iota(\overline{x}, \lambda) \in G\Omega_p(\tilde{X})$ by $\iota(\overline{x}, \lambda)_1(s,t) = (\overline{x}_1(s,t), \lambda_t - \lambda_s)$,

$$\iota(\overline{x}, \lambda)_2(s,t) = \Big(\overline{x}_2(s,t), \int_s^t \overline{x}_1(s,u) \otimes d\lambda_u, \int_s^t (\lambda_u - \lambda_s) \otimes \overline{x}_1(s, du), \int_s^t (\lambda_u - \lambda_s) \otimes d\lambda_u\Big).$$

This map $\iota : G\Omega_p(X) \times \mathrm{BV}(\mathbb{R}^N) \to G\Omega_p(\tilde{X})$ is also locally Lipschitz continuous.

For $\varepsilon \ge 0$, we set $\lambda^\varepsilon(t) := (a_1(\varepsilon)t, \dots, a_N(\varepsilon)t)$ and set $\Psi_\varepsilon : \mathrm{BV}(X) \to \mathrm{BV}(Y)$ by

$$\Psi_\varepsilon(h)_t := \Phi\big(\iota(\overline{h}, \lambda^\varepsilon)\big)_1(0,t), \qquad 0 \le t \le 1.$$

Then, $y := \Psi_\varepsilon(h)$ satisfies the following Y-valued usual ODE:

$$dy_t = \sigma(y_t) dh_t + \sum_{i=1}^N a_i(\varepsilon) b_i(y_t) dt \quad \text{with } y_0 = 0.$$

With these observations in mind, we define our Wiener functional X^ε as follows:

$$X^\varepsilon_t := \Phi\big(\iota(\overline{\varepsilon w}, \lambda^\varepsilon)\big)_1(0,t), \qquad 0 \le t \le 1.$$

Examples of X^ε include heat processes over loop spaces and solutions usual SDEs (1) over finite dimensional spaces or M-type 2 Banach spaces. (Because of Wong-Zakai's approximation theorem.)

In this talk we assume the following: Below, the Fréchet derivative of $BV(X), P(Y)$ is denoted by D、and that of Y is by denoted ∇.

(H1): F and G are real-valued bounded continuous functions defined on $P(Y)$.

(H2): The function $F_\Lambda := F \circ \Psi_0 + \| \cdot \|^2_{\mathcal{H}}/2$ defined on $\mathcal{H}$ attains its minimum at a unique point $\gamma \in \mathcal{H}$. For this γ, we write $\phi := \Psi_0(\gamma)$.

(H3): The functions F and G are $n+3$ and $n+1$ times Fréchet differentiable on a neighborhood $B(\phi)$ of $\phi \in P(Y)$, respectively. Moreover there exist positive constants $M_1, \dots, M_{n+3}$ such that

$$\begin{aligned} \big|D^k F(\eta)\big[y, \dots, y\big]\big| &\leq M_k \|y\|^k_{P(Y)}, \quad k = 1, \dots, n+3, \\ \big|D^k G(\eta)\big[y, \dots, y\big]\big| &\leq M_k \|y\|^k_{P(Y)}, \quad k = 1, \dots, n+1, \end{aligned}$$

hold for any $\eta \in B(\phi)$ and $y \in P(Y)$.

(H4): At the point $\gamma \in \mathcal{H}$, we consider the Hessian $A := D^2(F \circ \Psi_0)(\gamma)|_{\mathcal{H}\times\mathcal{H}}$. As a bounded self-adjoint operator on $\mathcal{H}$, the operator A is strictly larger than $-\mathrm{Id}_{\mathcal{H}}$ in the form sense. (Actually, A is Hilbert-Schmidt. By the min-max principle, it is equivalent to assume that infimum of all eigenvalues of A are strictly larger than -1.)

Our main result is as follows (Inahama-Kawabi [JFA, to appear]). We do not give the explicit forms of the constants α_n, $c(\gamma)$ here, because they are too long.

Theorem 1 *Under the assumptions* **(EX)**, **(H1)**–**(H4)**, *we have the following asymptotic expansion:*

$$\begin{aligned} &\mathbb{E}\Big[G(X^\varepsilon)\exp\big(-F(X^\varepsilon)/\varepsilon^2\big)\Big] \\ &= \exp\big(-F_\Lambda(\gamma)/\varepsilon^2\big)\exp\big(-c(\gamma)/\varepsilon\big)\cdot\big(\alpha_0 + \alpha_1\varepsilon + \cdots + \alpha_n\varepsilon^n + O(\varepsilon^{n+1})\big). \end{aligned} \tag{3}$$

The key of the proof is the (stochastic) Taylor expansion with respect to the topology of the rough path space given as below. (cf. Aida[preprints, 2005 & 2006]、Inahama-Kawabi [JFA, to appear])

$$\Phi(\iota(\overline{\gamma + \varepsilon w}, \lambda^\varepsilon))_1 = \phi + \varepsilon\phi^1(\overline{w})_1 + \varepsilon^2\phi^2(\overline{w})_1 + \cdots + \varepsilon^n\phi^n(\overline{w})_1 + O(\varepsilon^{n+1}).$$

Since this expansion is deterministic, it is natural to guess that the same mothod applies to asymptotic problems of other probability measures on $G\Omega_p(X)$.

Our method of the stochastic Taylor expansion is slightly different from Aida's method. He uses the derivative equation, whose coefficient is of course of linear growth. Since it is not known whether Lyons' continuity theorem holds or not for unbounded coefficients, he extends the continuity theorem for the case of the derivative equation in the first preprint. On the other hand, we use the method in Azencott's original paper [1982, LNM921] and we only need the continuity theorem for the given equation, whose coefficient is bounded. The price we have to pay is that notations and proofs may seem slightly long. However, the strategy of this method is quite simple and straight forward.

Yasunori Okabe
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Abstract For any strongly continuous resolvent $\{G_\alpha; \alpha > 0\}$ on a real Hilbert space H, we shall introduce a closed symmetric form $(\mathcal{E}(V), \mathcal{D}(\mathcal{E}(V))$ in a wide sense, to be called G-form, associated with the potential operator $V \equiv \lim_{\alpha\to 0} G_\alpha$ defined by Yosida transformation in Hunt's potential theory, besides the usual closed symmetric form $(\mathcal{E}(A), \mathcal{D}(\mathcal{E}(A))$, to be called D-form in this talk, associated with the infinitesimal generator A.

First, we shall characterize the G-form as a dual form of the D-form and the D-form as a dual form of the G-form through Legendre-Fenchel transformation in convex analysis.

Next, we shall find some relationships between the D-space $\mathcal{D}(\mathcal{E}(A))$ and the G-space $\mathcal{D}(\mathcal{E}(V))$, to be a domain of the D-form and the G-form, respectively. In particular, we shall obtain a fundamental relation between the resolvent $\{G_\alpha; \alpha > 0\}$ associated with the D-form and the resolvent $\{G^\bullet_\alpha; \alpha > 0\}$ associated with the G-form. Furthermore, we shall show that there exists a unitary operator $V^\bullet$ from the completed G-space onto the completed D-space, and characterise its inverse operator and its restriction to the space $\mathcal{D}(\mathcal{E}(V))$. Note that the restriction of the operator $V^\bullet$ to the space $\mathcal{D}(V)$ is the potential operator V. Moreover, we shall construct four kinds of resolvents by extending the resolvents $\{G_\alpha|_{\mathcal{D}(\mathcal{E}(A))}; \alpha > 0\}, \{G^\bullet_\alpha|_{\mathcal{D}(\mathcal{E}(A))}; \alpha > 0\}$ on the completed D-space and $\{G_\alpha|_{\mathcal{D}(\mathcal{E}(V))}; \alpha > 0\}, \{G^\bullet_\alpha|_{\mathcal{D}(\mathcal{E}(V))}; \alpha > 0\}$ on the completed G-space, respectively and then characterize them.

Finally, we shall consider the extended D-space and the extended G-space for a general case and investigate the problem concerning the equivalence of the non-degeneracy of seminorms and the completeness of them.

Conditioning Quadratic Wiener functionals and Plücker coordinates — with a new example

Setsuo Taniguchi (Kyushu Univ.)

Let $T > 0$ and $(\mathcal{W}, \mu)$ be the d-dimensional classical Wiener space over $[0,T]$, i.e., $\mathcal{W}$ is the Banach space of all $\mathbb{R}^d$-valued continuous functions defined on $[0,T]$ starting at the origin, and μ is the Wiener measure on $\mathcal{W}$. Denote by H the Cameron-Martin subspace of $\mathcal{W}$. Thinking of a symmetric Hilbert Schmidt operator $A : H \to H$ as a constant Wiener functional with values in the Hilbert space of Hilbert-Schmidt operators of H to itself, we define the quadratic Wiener functional associated with A by $Q_A = (\nabla^*)^2 A$, where ∇^* stands for the adjoint operator of the Malliavin gradient ∇. For linearly independent $\eta_1, \dots, \eta_M \in H$ and $N \leq M$, set $\boldsymbol{\eta}^{(N)} = (\nabla^* \eta_1, \dots, \nabla^* \eta_N)$. Investigated in this talk is the conditioned stochastic oscillatory integral

$$I_N(\zeta) = \int_{\mathcal{W}} e^{\zeta Q_A/2} \delta_0(\boldsymbol{\eta}^{(N)})\, d\mu, \quad \zeta \in \mathbb{C}$$

where δ_0 is the Dirac measure concentrated at $0 \in \mathbb{R}^N$ and $\delta_0(\boldsymbol{\eta}^{(N)})$ denotes the pull-back due to S. Watanabe. The exact expressions of the above integrals $I_N(\zeta)$, $1 \leq N \leq M$, will be given in terms of the Plücker coordinate of a point in the $(M, 2M)$-Grassmannian. Such correspondence was firstly pointed out and investigated by Hara-Ikeda [1] in the case of the classical and generalized Lévy areas. In this talk, we extend their observation to general cases with the help of the Jacobi field approach to quadratic Wiener functional introduced by Ikeda-Manabe [2].

We shall testify our generalization in a new quadratic Wiener functional which is obtained as the Malliavin derivative of the square norm of Brownian sample path. The Wiener functional attracts us since it determines the stationary point of the square norm. Some more detailed observations on the Wiener functional will be presented in the talk.

The talk is based on two recent papers [3,4].

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GENERALIZED POSITIVE CONTINUOUS ADDITIVE FUNCTIONALS OF MULTIDIMENSIONAL BROWNIAN MOTION AND THEIR ASSOCIATED REVUZ MEASURE

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(1) generalized PCAF. Let $(W_0^N, \mathcal{F}_t, P)$ be the N dimensional standard Wiener space, i.e., $W_0^N = \{W_t = (W_t^1, W_t^2, \ldots, W_t^N) : [0,\infty) \to \mathbb{R}^N | W_t$ is continuousand $W_0 = 0\}$, $\mathcal{F}_t = \sigma\{W_s; 0 \le s \le t\}$ and P is the standard Wiener measure. Let $\boldsymbol{D}_2^\gamma$ be the Meyer-Watanabe's Sobolev space. Let $\mathbb{R}_A^N$ be a subset of $\mathbb{R}^N$ satisfying $|\mathbb{R}^N \setminus \mathbb{R}_A^N| = 0$. We consider $A = \{A(t,x;W.); t \ge 0, x \in \mathbb{R}_A^N\} \subset \boldsymbol{D}_2^\gamma$.

Definition 1. (i) Let $\gamma < 0$. A is called a $\boldsymbol{D}_2^\gamma$ additive functional of the N dimensional Brownian motion if and only if (a) $A(t,x) = A(t,x;W.)$ is $\mathcal{F}_t$ measurable, (b) $A(0,x) = 0$ and

$$(c) \qquad A(t+s,x;W.) - A(t,x;W.) = A(s, x+W_t; (\theta_t W).)$$

in $\boldsymbol{D}_2^\gamma$, where $(\theta_t W)_s = W_{t+s} - W_t$.
(ii) $A(t,x)$ is positive if $\langle F, A(t,x)\rangle \ge 0$ for all $F(\in \boldsymbol{D}_2^{-\gamma}) \ge 0$.

Remark 1. $A(s,x+W_t;(\theta_t W).)$ is defined by $\int A(s,x+y;(\theta_t W).)\diamond_1 \delta(W_t - y)dy$, where $\diamond_1$ denotes the Wiener product (see [1]). If we assume Condition 1 below, then $A(s,x+W_t;(\theta_t W).) = \sum I_{\boldsymbol{n}}^{\theta_t W}(a_{\boldsymbol{n}}(t,x+W_t))$, where $A(t,x;W.) = \sum I_{\boldsymbol{n}}(a_{\boldsymbol{n}}(t,x))$.

Condition 1. $\int \|A(t,y)\|_{2,\gamma}^2 \mathrm{e}^{-\delta|y|^2} dy < \infty$ for all $\delta > 0$, where $\|\cdot\|_{2,\gamma}$ denotes the norm of $\boldsymbol{D}_2^\gamma$.

Proposition 1. *We assume $A(t,x)$ is continuous w.r.t. t in $\boldsymbol{D}_2^\gamma$. Let $\Delta = \{0 = t_0 < t_1 < \cdots < t_n < t_{n+1} = t\}$ be a partition of $[0,t]$, and put $|\Delta| = \max|t_{i+1} - t_i|$. Then we obtain*

$$\lim_{|\Delta|\to 0} \sum_{i=0}^n a_0(t_{i+1} - t_i, x + W_{t_i}) = A(t,x)$$

in $\boldsymbol{D}_2^\gamma$.

(2) Revuz measure associated to generalized PCAF. Let A be a $\boldsymbol{D}_2^\gamma$ positive continuous additive functional (abbreviated $\boldsymbol{D}_2^\gamma$ PCAF). For all $f \in \mathcal{D}$

$$\int_0^t \langle f(x+W_s), dA_s(x)\rangle = \lim_{|\Delta|\to 0} \sum_{i=0}^n \langle f(x+W_{t_i}), A(t_{i+1},x) - A(t_i,x)\rangle$$

is well defined. Moreover, under Condition 1,

$$f(\in \mathcal{D}) \mapsto \int_{\mathbb{R}^N} \int_0^t \langle f(x+W_s), dA_s(x)\rangle dx \in \mathcal{D}'.$$

Definition 2. Assume Condition 1. The Revuz measure ν_A associated to $\boldsymbol{D}_2^\gamma$ PCAF A is the measure on $\mathbb{R}^N$ such that

$$\int_{\mathbb{R}^N} f(x)\nu_A(dx) = \int_{\mathbb{R}^N}\int_0^1 \langle f(x+W_s), dA_s(x)\rangle dx$$

for all $f \in \mathcal{D}$.

Proposition 2. *Under Condition 1*

$$\int_{\mathbb{R}^N} f(x)\nu_A(dx) = \lim_{|\Delta|\to 0}\sum_{i=0}^{n}\int_{\mathbb{R}^N} f(x)a_0(t_{i+1}-t_i, x)dx,$$

Δ *denoting a partition of* $[0,1]$.

(3) Local time representation of generalized PCAF.

Condition 2. For all $\eta > 0$ small enough and for all $\delta > 0$,

$$\int |y-x|^{2-N-\eta}\mathrm{e}^{-\delta|y-x|^2}\mu(dy) < \infty.$$

Theorem 1. *Let* $A = \{A(t,x;W.); t \geq 0, x \in \mathbb{R}_A^N\}$ *be a* $\boldsymbol{D}_2^\gamma$ *PCAF satisfying Condition 1. Assume* ν_A *satisfies Condition 2. Then it holds that* $\int L(t,y-x)\nu_A(dy)$ *exists in* $\boldsymbol{D}_2^\alpha$ $(\alpha < 1-N/2)$ *and that*

$$A(t,x) = \int L(t,y-x)\nu_A(dy),$$

where $L(t,z)$ *denotes the local time of* W *at* z.

(4) generalized PCAF corresponding to Radon measure. Let $T \in \mathcal{D}'$ be a positive distribution and μ_T be the corresponding Radon measure. Let $\alpha < 1-N/2$. Then we obtained a $\boldsymbol{D}_2^\alpha$ PCAF $A_T(t,x)$ corresponding to T under Condition 2 ([2]). Applying Mehler's formula, we have the followings:

Theorem 2. *Assume* μ_T *satisfies Condition 2. Then* $A_T(t,x) \in \boldsymbol{D}_2^{-\beta}$ *if and only if*

$$\int_0^\infty \iint \int_0^t \int_0^s \mathrm{e}^{-r} r^{\beta-1} p_N(s-\mathrm{e}^{-2r}u, y-\mathrm{e}^{-r}z-(1-\mathrm{e}^{-r})x) \times p_N(u, z-x)duds\mu_T(dz)\mu_T(dy)dr < \infty.$$

Theorem 3. *Assume* μ_T *satisfies Condition 2. If*

$$\iint |y-z|^{2-N-\eta}|z-x|^{2-N-\eta}\mathrm{e}^{-\delta|y-z|^2}\mathrm{e}^{-\delta|z-x|^2}\mu_T(dy)\mu_T(dz) < \infty,$$

then $A_T(t,x) \in L^2(P)$.

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APPLICATIONS OF THE MALLIAVIN CALCULUS OF BISMUT TYPE WITHOUT PROBABILITY.

Rémi Léandre.

The talk is divided in 4 parts:

-)In the first part we translate in semi-group theory Bismut way of the Malliavin Calculus.

-)In the second part, we translate in semi-group theory our proof of Varadhan estimates, lower bound, got a lot time ago by us by using the Malliavin Calculus.

-)In the third part, we translate in semi-group theory Wong-Zakai approximation of a diffusion and the proof of the positivity result got by using Bismut's procedure a long time ago by Ben Arous and us.

-)In the fourth part, we translate in semi-group theory the division method and the results got a long time ago by us by using the Malliavin Calculus.

Witten Laplacians on pinned path groups

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Let G be a simply connected and connected compact Lie group. Let $a \in G$. We denote the unit element of G by e. We consider a pinned path group

$$P_{e,a}(G) = C([0,1] \to G \mid \gamma(0) = e, \gamma(1) = a). \tag{1}$$

Let $\lambda > 0$ and $\nu_{\lambda,a}$ be the pinned Brownian motion measure on $P_{e,a}(G)$ such that

$$\begin{aligned}
&\nu_{\lambda,a}\left(\{\gamma \in P_{e,a}(G) \mid \gamma(t_1) \in A_1, \ldots, \gamma(t_{n-1}) \in A_{n-1}\}\right) \\
&\quad = p\left(\lambda^{-1}, e, a\right)^{-1} \int_{G^n} \prod_{i=1}^{n-1} p\left(\lambda^{-1}\left(t_i - t_{i-1}\right), x_{i-1}, x_i\right) 1_{A_i}(x_i) \cdot p\left(\lambda^{-1}\left(1 - t_{n-1}\right), x_{n-1}, a\right) \\
&\qquad\qquad dx_1 \cdots dx_{n-1},
\end{aligned}$$

where $0 = t_0 < t_1 < \cdots < t_{n-1} < 1$, $x_0 = e$ and $A_i \subset G$. $p(t, x, y)$ denotes the heat kernel of $e^{(t/2)\Delta}$.

We are interested in the following Witten Laplacian.

Definition 1 *Let d be a exterior derivative on $P_{e,a}(G)$. Let $d^*_{\nu_{\lambda,a}}$ denote the adjoint operator of d on $L^2(\wedge T^* P_{e,a}(G), d\nu_{\lambda,a})$. Set $\square_{\lambda,a} = -\left(dd^*_{\nu_{\lambda,a}} + d^*_{\nu_{\lambda,a}} d\right)$. For an open set Ω in $P_{e,a}(G)$, set*

$$E_{Dir,1}(\lambda, \Omega) = \inf\left\{ (-\square_{\lambda,a}\alpha, \alpha)_{L^2(\nu_{\lambda,a})} \;\middle|\; \alpha \text{ is a smooth 1-form and } \alpha|_{\partial\Omega} = 0 \right\}.$$

Definition 2 *For $\gamma, \eta \in P_{e,a}(G)$, define*
$d(\gamma, \eta) = \max_{0 \le t \le 1} d(\gamma(t), \eta(t))$. Also let $E(\gamma) = \frac{1}{2}\int_0^1 |\dot{\gamma}(t)|^2 dt$ and

$$\Omega_L = \left\{\gamma \in P_{e,a}(G) \mid \sqrt{E(\gamma)} \le L\right\} \tag{2}$$

$$B_\varepsilon(\eta) = \{\gamma \in P_{e,a}(G) \mid d(\gamma, \eta) < \varepsilon\} \tag{3}$$

$$\Omega_{L,\varepsilon} = \left\{\gamma \in P_{e,a}(G) \mid \text{there exists } \eta \in \Omega_L \text{such that } \gamma \in B_\varepsilon(\eta)\right\}. \tag{4}$$

Also we define for $0 < \alpha < 1$,

$$\|\gamma\|_\alpha = \sup_{0 \le s,t \le 1} \frac{d(\gamma(t), \gamma(s))}{|t - s|^\alpha}. \tag{5}$$

Remark 3 *Let $R > 0$. Set*

$$D_{\alpha,R} = \{\gamma \in P_{e,a}(G) \mid \|\gamma\|_\alpha < R\}.$$

Then for any small $\varepsilon > 0$, there exists $L > 0$ such that $D_{\alpha,R} \subset \Omega_{L,\varepsilon}$.

The following is one of our main result.

Theorem 4 *Suppose that a is outside the cut-locus of e. We denote the all geodesics connecting e and a by $\{c_i\}_{i=1}^\infty \subset P_{e,a}(G)$. Also we denote the all eigenvalues of $(\nabla^2 E)(c_i)$ by $\{\xi_j(c_i) \mid j = 1, 2, \ldots\}$. Let Ω be an open subset of $P_{e,a}(G)$. Assume that there exists a sufficeintly small $\varepsilon > 0$ and positive L such that $\Omega \subset \Omega_{L,\varepsilon}$ and $\partial\Omega$ does not contain any geodesics. Then it holds that*

$$\lim_{\lambda\to\infty} \frac{E_{Dir,1}(\lambda, \Omega)}{\lambda} = \min\{\theta_1(c_i) \mid c_i \in \Omega\},$$

where

$$\theta_1(c_i) = \inf_{j\geq 1} \left(\max(\xi_j(c_i), 0) + \sum_{\{k\neq j, \xi_k(c_i)<0\}} |\xi_k(c_i)| \right).$$

Further, it holds that $\theta_1(c_i) > 0$ for all $i \geq 1$.

Remark 5 *When $G = SU(n, \mathbb{C}), SO(n)$, it holds that*

$$\theta_1(c_i) \geq \frac{l(c_i)}{2\pi \dim G} - C_1 \to +\infty \quad (i \to \infty). \tag{6}$$

where $l(c_i)$ denotes the length of the geodesic c_i and C is a constant.

The main theorem can be proved by applying the following log-Sobolev inequality (with a potential function). Below, a is not necessarily outside the cut-locus of e.

Theorem 6 *There exist constants $C_1, C_2 > 0$ such that for any sufficiently large $\lambda > 0$ and $f \in \mathfrak{F}C_b^\infty(P_{e,a}(G))$, it holds that*

$$\int_{P_{e,a}(G)} f^2(\gamma) \log\left(\frac{f^2(\gamma)}{\|f\|^2_{L^2(\nu_{\lambda,a})}}\right) d\nu_{\lambda,a}(\gamma) \leq \frac{2}{\lambda}\left(1 + \frac{C_1}{\lambda}\right) \mathcal{E}_{\lambda,V_{\lambda,a}}(f, f), \tag{7}$$

where

$$\mathcal{E}_{\lambda,V_{\lambda,a}}(f, f) = \int_{P_{e,a}(G)} |(\nabla f)(\gamma)|_H^2 d\nu_{\lambda,a} + \int_{P_{e,a}(G)} \lambda^2 V_{\lambda,a}(\gamma) f(\gamma)^2 d\nu_{\lambda,a}, \tag{8}$$

$$V_{\lambda,a}(\gamma) = \frac{1}{4}\left\{|b(1)|^2 + \frac{2}{\lambda}\log\left(\lambda^{-d/2} p(1/\lambda, e, a)\right)\right\} + \frac{C_2}{\lambda}\left\{1 + |b(1)|^2 + \left(\int_0^1 |b(s)|ds\right)^2\right\}.$$