

ASYMPTOTIC COMPACTNESS AND ABSORBING SETS FOR STOCHASTIC BURGERS EQUATIONS DRIVEN BY SPACE TIME WHITE NOISE AND FOR SOME 2D STOCHASTIC NAVIER-STOKES EQUATIONS ON CERTAIN UNBOUNDED DOMAINS

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ABSTRACT

In the last decade there has been a growing interest in the ergodic properties of infinite dimensional systems governed by stochastic partial differential equations. In particular existence of attractors for 2-D Navier Stokes Equations in bounded domains both driven by real and additive noise has been established, see e.g. [2], [5] and [10]. Recently in a joint work with Y. Li we have generalised the results from [5] and [10] to the case of unbounded domains. We observed there that the method of asymptotical compactness used by us should work also for equations in bounded domains with much rougher noise than the original methods could handle. The main motivation of this talk, which is mainly based on a very recent paper [1], is to show that this is indeed the case for the 1-D stochastic Burgers equations with additive space-time white noise. Even for readers mainly interested in the Navier-Stokes equations it could be useful to study the Burgers equations case. Contrary to some recent works on stochastic Burgers equations, see [6] and references therein, our approach is very similar to the approach we use for the NSEs in [3]. The second motivation is to show that it also works for certain special form of 2-D stochastic NSEs with multiplicative noise. In fact, using generalisation of a recent result [4] to bounded and unbounded domains, we show the existence of a compact invariant set for such problems. Full proofs of the results presented in this section will be published elsewhere. We should make it clear that we only study functional version of Stochastic NSEs. The main tool is a new notion of asymptotically compact random dynamical system. It is motivated by works of Ladyzhenskaya [8], Ghidaglia [7] and Rosa [9].

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