Stochastic Analysis

This conference is held as a part of the RIMS Research Project 2023: Stochastic Processes and Related Fields.

Date : November 6 (Monday) – November 9 (Thursday), 2023
Place : Room 420 of RIMS, Kyoto university
Webpage : `https://www.math.kyoto-u.ac.jp/~kusuoka/workshop/RIMS2023/`

Organizers : Yuzuru Inahama (Kyushu University)
Hiroshi Kawabi (Keio University)
Seiichiro Kusuoka (Kyoto University)
Song Liang (Waseda University)

—Time table—

Nov. 6 (Mon)	Nov. 7 (Tue)	Nov. 8 (Wed)	Nov. 9 (Thu)
10:10–10:20 Opening	9:30–10:10 Fumiya Okazaki	9:30–10:10 (Free discussion)	9:30–10:10 Kohei Suzuki
10:20–11:00 Dan Crisan	10:20–11:00 Wei Liu	10:20–11:00 Nobuaki Naganuma	10:20–11:00 Lorenzo Dello Schiavo
11:10–11:50 Arturo Kohatsu-Higa	11:10–11:50 Masahisa Ebina	11:10–11:50 Makoto Nakashima	11:10–11:50 Kazuhiro Kuwae
11:50–13:30 (Lunch break)	11:50–13:30 (Lunch break)	11:50–13:30 (Lunch break)	11:50–13:30 (Lunch break)
13:30–14:10 Ryuya Namba	13:30–14:10 Dejun Luo	13:30–14:10 Hirotatsu Nagoji	13:30–14:10 Hirotaka Kai
14:20–15:00 Dai Taguchi	14:20–15:00 Toyomu Matsuda	14:20–15:00 Kouhei Matsuura	14:20–15:00 Atsushi Takeuchi
15:20–16:00 Ana Bela Cruzeiro	15:20–16:00 Ismael Bailleul	15:20–16:00 Jean Claude Zambrini	15:20–16:00 Yushi Hamaguchi
16:10–16:50 Toshio Mikami	16:10–16:50 Bin Xie	16:10–16:50 Shigeki Aida	16:10–16:50 Josef Teichmann

—Program—

November 6 (Monday)

10:10–10:20 Opening

10:20–11:00 Dan Crisan (Imperial College London)
Analytical properties for stochastic transport models - an overview

11:10–11:50 Arturo Kohatsu-Higa (Ritsumeikan University)
Probabilistic representation for the derivative of a killed process: The multi-dimensional case

11:50–13:30 Lunch break

13:30–14:10 Ryuya Namba (Kyoto Sangyo University)
Iterates of Bernstein-type operators and some diffusions in population genetics

14:20–15:00 Dai Taguchi (Kansai University)
Besov regularity of the density function for SDEs with super-linearly growing coefficients

15:20–16:00 Ana Bela Cruzeiro (University of Lisbon)
On a pathwise stochastic control problem

16:10–16:50 Toshio Mikami (Tsuda University)
Stochastic optimal transport with at most quadratic growth cost

November 7 (Tuesday)

9:30–10:10 Fumiya Okazaki (Tohoku University)
Description of fractional harmonic maps via discontinuous martingales on Riemannian submanifolds

10:20–11:00 Wei Liu (Jiangsu Normal University)
Well-posedness and Asymptotics of MVSPDEs

11:10–11:50 Masahisa Ebina (Kyoto University)
Ergodicity and central limit theorems for stochastic wave equations in high dimensions

11:50–13:30 Lunch break

13:30–14:10 Dejun Luo (Chinese Academy of Sciences)
Limit theorems for stochastic inviscid Leray-α model with transport noise

14:20–15:00 Toyomu Matsuda (EPFL)
Level crossings of fractional Brownian motion

15:20–16:00 Ismael Bailleul (University of Brest)
Phi43 measures on compact Riemannian 3-manifolds

16:10–16:50 Bin Xie (Shinshu University)
Global solvability and stationary solutions of singular quasilinear SPDEs

November 8 (Wednesday)

9:30–10:10 Free discussion

10:20–11:00 Nobuaki Naganuma (Kumamoto University)
An approach to asymptotic error distributions of rough differential equations

11:10–11:50 Makoto Nakashima (Nagoya University)
Stochastic quantization of the three dimensional polymer measure

11:50–13:30 Lunch break

13:30–14:10 Hirotatsu Nagoji (Kyoto University)
Normalizability of the Gibbs measures associated with multivariate version of $P(\Phi)_2$ model

14:20–15:00 Kouhei Matsuura (Tsukuba University)
Discrete approximation of reflected Brownian motions by Markov chains on partitions of domains

15:20–16:00 Jean Claude Zambrini (University of Lisbon)
Geometric Mechanics of the processes solving Schrödinger's problem

16:10–16:50 Shigeki Aida (University of Tokyo)
Asymptotics of lowlying Dirichlet eigenvalues of Witten Laplacians on domains in pinned path groups

November 9 (Thursday)

9:30–10:10 Kohei Suzuki (Durham University)
Curvature bound of the Dyson Brownian motion

10:20–11:00 Lorenzo Dello Schiavo (Institute of Science and Technology Austria)
Liouville Quantum Gravity in Even Dimension

11:10–11:50 Kazuhiro Kuwae (Fukuoka University)
Hess-Schrader-Uhlenbrock inequality for the heat semigroup on differential forms over Dirichlet spaces tamed by distributional curvature lower bounds

11:50–13:30 Lunch break

13:30–14:10 Hirotaka Kai (Kyoto University)
Estimates for the radial part of jump-diffusion processes on manifolds and their applications

14:20–15:00 Atsushi Takeuchi (Tokyo Woman's Christian University)
Wasserstein distance on solutions to stochastic differential equations with jumps

15:20–16:00 Yushi Hamaguchi (Osaka University)
Weak well-posedness of stochastic Volterra integral equations

16:10–16:50 Josef Teichmann (ETH Zurich)
Ergodic robust maximization of asymptotic growth under stochastic factors

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「ディリクレ形式に基づく確率解析の研究—空間構造と特異性の解明—」
Principal Investigator: Masanori Hino (Kyoto University)
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「複雑な系の上の確率過程と確率解析の展開」
Principal Investigator: Takashi Kumagai (Waseda University)
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「確率解析の新展開」
Principal Investigator: Yuzuru Inahama (Kyushu University)
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「繰り込みを伴う方程式と確率解析」
Principal Investigator: Seiichiro Kusuoka (Kyoto University)
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「幾何学的視点を融合した無限次元空間上の確率解析の研究」
Principal Investigator: Hiroshi Kawabi (Keio University)

—Abstracts—

Analytical properties for stochastic transport models - an overview

Dan Crisan
(Imperial College London)

In this talk I will present some recent results related to existence and uniqueness of the solution of some stochastic PDEs for geophisycal fluid dynamics models. This is joint work with O. Lang, D Goodair and É. Mémin.

Relevant papers:

[1] O Lang, D. Crisan, Dan; É. Mémin, Analytical properties for a stochastic rotating shallow water model under location uncertainty, J. Math. Fluid Mech. 25, no. 2, 2023.

[2] D Crisan, O Lang, Well-posedness Properties for a Stochastic Rotating Shallow Water Model, Journal of Dynamics and Differential Equations, 1-31, 6, 2023

[3] D Goodair, D Crisan, O Lang, Existence and uniqueness of maximal solutions to SPDEs with applications to viscous fluid equations, Stochastics and Partial Differential Equations: Analysis and Computations, 1-64, 3, 2023.

[4] O Lang, D Crisan, Well-posedness for a stochastic 2D Euler equation with transport noise, Stochastics and Partial Differential Equations: Analysis and Computations 11, 13, 2023.

Probabilistic representation for the derivative of a killed process: The multi-dimensional case

Arturo Kohatsu-Higa
(Ritsumeikan University)

In this presentation, I will explain some results (in preparation) that have been obtained with F. Antonelli (U. L'Aquila) on the representation of derivative of the killed diffusion semigroup. This representation is based on a exponential process solution of a linear stochastic equation which contains a local time term. The result can be read as an extension of the classical results of derivatives of flow diffusions. In fact, recall the classical formula for $f, b, \sigma \in C_b^1$:

$$\begin{aligned}
X_t =& x + \int_0^t b(X_s)ds + \int_0^t \sigma^k(X_s)dW_s^k \\
\nabla_x P_T f(x) =& \nabla_x \mathbb{E}\left[f(X_T)\right] = \mathbb{E}\left[f'(X_T)\nabla_x X_T\right] \\
\nabla_x X_t =& 1 + \int_0^t \mathbf{D}b(X_s)\nabla_x X_s ds + \int_0^t \mathbf{D}\sigma^k(X_s)\nabla_x X_s dW_s^k
\end{aligned}$$

We extend the above formula in the following sense:

$$\nabla_x P_T f(x) = \nabla_x \mathbb{E}\left[f(X_T)1_{(\tau>T)}\right] = \mathbb{E}\left[f'(Y_T)\Psi_T\right].$$

Here $\tau \equiv \tau^x := \inf\{s > 0; X_s \in \nabla D\}$ where D is a smooth domain and Y is the solution of an obliquely reflected stochastic differential equation with coefficients b and σ. Finally Ψ is the solution of a linear equation with diffusion coefficient $\mathbf{D}\sigma^k(Y_s)\Psi_s$, drift coefficient $\mathbf{D}b(X_s)\Psi_s$ and local time terms which have some geometrical meaning which will be discussed in the presentation. Furthermore Ψ presents jumps everytime that Y touches the boundary. These effects are "dual" to similar results about pathwise differentiation of reflected processes. These results extend the previously studied situations of the one dimensional case and the half space case with no correlation at the boundary which were obtained in joint work with D. Crisan (Imperial College).

Iterates of Bernstein-type operators and some diffusions in population genetics

Ryuya Namba
(Kyoto Sangyo University)

The Bernstein operator is a positive linear operator on the Banach space of continuous functions on $[0, 1]$, which is used to show the celebrated Weierstrass approximation theorem from a probabilistic perspective. In this talk, we introduce an extension of the Bernstein operator to the d-dimensional cases and discuss some limit theorems for the iterates of the operator. As the limit, we capture the d-dimensional Wright–Fisher diffusion with mutation which is well-studied in population genetics. Some further possible directions of these limit theorems including infinite-dimensional cases are discussed as well. Based on a joint work with Takatoshi Hirano.

Besov regularity of the density function for SDEs with super-linearly growing coefficients

Dai Taguchi
(Kansai University)

In 2010, Fournier–Printems [2] introduced a simple method for proving the existence of the density function of the time marginals of one-dimensional SDEs with linear growth (and non-smooth) coefficients. Debussche–Fournier [1] and Romito [4] extended this method to multi-dimensional case, and proved that the density function belongs to some Besov space. Their approach is based on"the one-step Euler–Maruyama scheme". On the other hand, Hutzenthalerm–Jentzen–Kloeden [3] showed that if the coefficients of SDE grow super-linearly, then the standard Euler–Maruyama scheme does not converge to a solution of the equation. In order to approximate a solution of these SDEs, several "tamed Euler–Maruyama schemes" are proposed. In particular, Sabanis [5] provided the rate of convergence for the scheme with super-linearly growing coefficients.

In this talk, inspired by these previous research, we prove the Besov regularity of the density function for a class of SDEs with super-linearly growing coefficients (for example, the stochastic Ginzburg–Landau equation in the theory of super conductivity, the Heston-3/2 volatility model in mathematical finance, and the stochastic Suart–Landau oscillator (neuronal model, hypoelliptic case)). Our approach is based on "the one-step tamed Euler–Maruyama scheme".

This talk is based on joint work with Tsukasa Moritoki (Okayama university).

References

[1] Debussche, A. and Fournier, N. Existence of densities for stable-like driven SDE's with Hölder continuous coefficients. *J. Funct. Anal.* **264**(8) 1757–1778 (2013).

[2] Fournier, N. and Printems, J. Absolute continuity of some one-dimensional processes. *Bernoulli* **16** 343–360 (2010).

[3] Hutzenthaler, M., Jentzen, A. and Kloeden, P. E. Strong convergence of an explicit numerical method for SDEs with nonglobally Lipschitz continuous coefficients. *Ann. Appl. Probab.* **22**(4) 1611–1641 (2012).

[4] Romito, M. A simple method for the existence of a density for stochastic evolutions with rough coefficients. *Electorn. J. Probab.* **23**(113) 1–43 (2018).

[5] Sabanis, S. Euler approximations with varying coefficients : the case of superlinearly growing diffusion coefficients. *Ann. Appl. Probab.* **25**(4) 2083–2105 (2016).

On a pathwise stochastic control problem

Ana Bela Cruzeiro

(University of Lisbon)

We consider a pathwise stochastic optimal control problem and derive the associated Hamilton-Jacobi-Bellman stochastic partial differential equation. We show that the value process is the unique solution of this equation, in the viscosity sense. This process will not be necessarily adapted.

This is a joint work with N. Bhauryal (Univ. of Lisbon) and C. Oliveira (NTNU, Norway).

Stochastic optimal transport with at most quadratic growth cost

Toshio Mikami
(Tsuda University)

In this talk, we consider a class of stochastic optimal transport, SOT for short, with given two endpoint marginals in the case where a cost function exhibits at most quadratic growth. We first give the upper and lower estimates, the short– and long–time asymptotics, and the zero–noise limits of SOT. As a by–product, we characterize the finiteness of the value function of SOT by that of the Monge–Kantorovich problem with the same two endpoint marginals. As an application, we show the existence of a continuous semimartingale, with given initial and terminal distributions, of which the drift vector is rth integrable for $r \in [1, 2)$. We also show that the value function of SOT is equal to zero or infinity in the case where a cost function exhibits less than linear growth. We discuss the same problem for Schrödinger's problem where $r = 2$. This paper is a continuation of our previous work.

Description of fractional harmonic maps via discontinuous martingales on Riemannian submanifolds

Fumiya Okazaki
(Tohoku University)

Harmonic maps are critical points of the energy functional defined for maps between Riemannian manifolds. It is known that these maps can be characterized through Brownian motion and martingales on manifolds. In this talk, we will consider a probabilistic characterization of harmonic maps with respect to non-local Dirichlet forms. The most typical examples of such harmonic maps are fractional harmonic maps, which are defined as harmonic maps with respect to the fractional Laplacian. To begin with, I will recall the definition of discontinuous martingales on Riemannian submanifolds. These are special cases of martingales considered in Picard (1991). Then I will elaborate on a characterization of harmonic maps for non-local Dirichlet forms via stochastic processes. In particular, from the characterization, we can obtain discontinuous martingales on Riemannian submanifolds by mapping symmetric stable processes by fractional harmonic maps. As simple applications of this approach, I will provide the Liouville-type theorem for fractional harmonic maps with values on a sphere. I will also explain the relation between the singularities of martingales on manifolds and those of harmonic maps.

Well-posedness and Asymptotics of MVSPDEs

Wei Liu
(Jiangsu Normal University)

In this talk we mainly present some well-posedness and asymptotic results for a class of Mckean-Vlasov SPDEs and multi-scale stochastic systems, in particular, we show the averaging principle, large deviations principle and central limit type theorem for multiscale SPDEs and MVS(P)DEs.

Ergodicity and central limit theorems for stochastic wave equations in high dimensions

Masahisa Ebina
(Kyoto University)

This talk considers a stochastic wave equation in spatial dimensions higher than three. The driving noise is assumed to be a Gaussian noise that is white in time and has some spatial correlation. I will discuss the difficulties in considering high-dimensional stochastic wave equations and explain how to apply the Malliavin calculus tools to study the solution's ergodicity and fluctuations. Specifically, under some conditions for the spatial correlation of noise, we show that the solution process is ergodic under spatial shifts and that its spatial average converges to the standard normal distribution.

Limit theorems for stochastic inviscid Leray-α model with transport noise

Dejun Luo
(Chinese Academy of Sciences)

We consider the stochastic inviscid Leray-α model on the torus driven by transport noise. Under a suitable scaling of the noise, we prove that the weak solutions converge, in some negative Sobolev spaces, to the unique solution of the deterministic viscous Leray-α model. Interpreting such limit result as a law of large numbers, we study the underlying central limit theorem and provide an explicit convergence rate. This talk is based on a joint work with Doctor Bin Tang.

Level crossings of fractional Brownian motion

Toyomu Matsuda
(École polytechnique fédérale de Lausanne)

The talk is based on joint work [Das+23] with Purba Das (King's College London), Rafał Łochowski (Warsaw) and Nicolas Perkowski (FU Berlin). We consider level crossings of fractional Brownian motion. Our main result is that the number of ε-level crossings at 0, after appropriate normalisation, converges to the local time at 0 multiplied by some constant $\mathfrak{c}_H$. Our key tool is the shifted stochastic sewing, recently obtained by Perkowski and the speaker [MP23]. I will also report an interesting conjecture on the constant $\mathfrak{c}_H$, which seems to capture non-Markovian nature of the fractional Brownian motion.

References

[Das+23] P. Das, R. Lochowski, T. Matsuda, and N. Perkowski. Level crossings of fractional Brownian motion. 2023. arXiv: 2308.08274 [math.PR].

[MP23] T. Matsuda and N. Perkowski. An extension of the stochastic sewing lemma and applications to fractional stochastic calculus. 2023. arXiv: 2206.01686 [math.PR].

Phi43 measures on compact Riemannian 3-manifolds

Ismaël Bailleul
(University of Brest)

I will explain in this talk what are the difficulties one has to bypass to construct the Phi43 measure on a 3-dimensional Riemannian manifold. Its construction provides the first example of a non-perturbative 'Euclidean' quantum field theory in a curved background.

Global solvability and stationary solutions of singular quasilinear SPDEs

Bin Xie
(Shinshu University)

In this talk, we consider singular quasilinear stochastic PDEs with spatial white noise as a potential over 1-dimensional torus. Such singular stochastic PDEs are relative to the study of the hydrodynamic scaling limit of a microscopic interacting particle system in a random environment. Under some sufficient conditions on coefficients and noise, we study the global existence of solutions in paracontrolled sense, and we also show the convergence of the solutions to its stationary solutions as time goes to infinity. We first introduce a proper energy functional and then use the approach based on energy inequality and Poincaré inequality of it in our proofs. This talk is based on the joint work with T. Funaki.

An approach to asymptotic error distributions of rough differential equations

Nobuaki Naganuma
(Kumamoto University)

Asymptotic error distributions of solutions to stochastic differential equations driven by fractional Brownian motion (fBm) has been studied in several situations, which are classified by dimension and roughness of the driving fBm ([6, 3, 5, 1, 4]). In this talk we will show results of the problem in the case that fBm is multi-dimensional and rough. One of difficulties of this problem is expression of the error term. We introduce certain interpolation process between the solution and the approximation to overcome this difficulty. This talk is based on the joint work with Shigeki Aida ([2])

References

[1] S. Aida and N. Naganuma. Error analysis for approximations to one-dimensional SDEs via the perturbation method. *Osaka J. Math.*, 57(2):381–424, 2020.

[2] S. Aida and N. Naganuma. An approach to asymptotic error distributions of rough differential equations. arXiv:2302.03912, 2023.

[3] Y. Hu, Y. Liu, and D. Nualart. Rate of convergence and asymptotic error distribution of Euler approximation schemes for fractional diffusions. *Ann. Appl. Probab.*, 26(2):1147–1207, 2016.

[4] Y. Hu, Y. Liu, and D. Nualart. Crank-Nicolson scheme for stochastic differential equations driven by fractional Brownian motions. *Ann. Appl. Probab.*, 31(1):39–83, 2021.

[5] Y. Liu and S. Tindel. First-order Euler scheme for SDEs driven by fractional Brownian motions: the rough case. *Ann. Appl. Probab.*, 29(2):758–826, 2019.

[6] N. Naganuma. Asymptotic error distributions of the Crank–Nicholson scheme for SDEs driven by fractional Brownian motion. *J. Theoret. Probab.*, 28(3):1082–1124, 2015.

Stochastic quantization of the three dimensional polymer measure

Makoto Nakashima
(Nagoya University)

We consider the Edwards model which describes a self-repulsive polymer. It is formally defined by

$$\nu_\lambda(\mathrm{d}\omega) = \frac{1}{Z_\lambda} \exp\left(-\lambda J(\omega)\right) \nu_0(\mathrm{d}\omega)$$

for $\lambda > 0$, where ν_0 denotes the Wiener measure, $J(\omega) = \int_0^1 \int_0^1 \delta_0(\omega_t - \omega_s)\mathrm{d}s\mathrm{d}t$ is a self-intersection local time, and Z_λ is a normalizing constant. ν_λ has been mathematically constructed for $d \leq 3$ in 1980s and it is known that ν_λ is singular with respect to ν_0 for $\lambda > 0$ when $d = 3$.

The stochastic quantization of ν_λ for $d = 2$ was studied by Albeverio, Hu, Röckner, Zhou but the case for $d = 3$ has been left. We talk about the stochastic quantization of ν_λ for $d = 3$ and mutually absolute continuity of ν_λ and $\nu_\lambda \circ \tau_h^{-1}$, where $\tau_h : \omega \mapsto \omega + h$ is a translation of a sample path ω by a continuous function h.

This talk is based on the joint work with Sergio Albeverio, Seiichiro Kusuoka, Song Liang.

Normalizability of the Gibbs measures associated with multivariate version of $P(\Phi)_2$ model

Hirotatsu Nagoji
(Kyoto University)

We consider the Gibbs measures associated with multivariate version of $P(\Phi)_2$ model on the two dimensional torus. We observe the (non-)normalizability of the measures by the variational method introduced by Barashkov and Gubinelli. We also consider some variants of the model such as the Gibbs measures tamed by Wick ordered L^2 norm.

Discrete approximation of reflected Brownian motions by Markov chains on partitions of domains

Kouhei Matsuura
(University of Tsukuba)

In this talk, we consider discrete approximation of reflected Brownian motions on domains in Euclidean space. Our approximation is given by a sequence of Markov chains on partitions of the domain, where we allow uneven or random partitions. We provide sufficient conditions for the weak convergence of the Markov chains. This is a joint work with Masanori Hino (Kyoto University) and Arata Maki.

Geometric Mechanics of the processes solving Schrödinger's problem

Jean Claude Zambrini
(University of Lisbon)

The community of Optimal Transportation rediscovered recently "Schrödinger's (variational) problem" (1931-32) for diffusions, mostly inspired by T. Mikami's series of works initiated 20 years ago ([2]).

We shall describe the (stochastic) dynamical theory of those critical processes. They admit Lagrangian, Hamiltonian and Noetherian descriptions. As suggested originally by Schrödinger, their dynamical properties are very reminiscent of the ones expected for corresponding (but inexistent) quantum "processes". Using the terminology of modern mathematical quantum field theory, Schrödinger's inspired dynamical theory provides historically the first Euclidean approach to all aspects of elementary Q.M.

We shall emphasize particularly the Hamiltonian approach, developed recently by Qiao Huang and myself ([1]), using tools of L. Schwartz and P.A. Meyer in their "Second-order approach" to stochastic differential geometry.

[1] Q. Huang, J.-C. Z., *From second-order differential geometry to stochastic geometric mechanics,* JNLS 33 (2023), Open access

[2] T. Mikami, *Stochastic Optimal Transportation (Stochastic control with fixed marginals),* Springer 2021

Asymptotics of lowlying Dirichlet eigenvalues of Witten Laplacians on domains in pinned path groups

Shigeki Aida
(University of Tokyo)

In mathematical physics, there are probability measures ν_λ on infinite dimensional spaces X which are formally written as the path integral $d\nu_\lambda = Z_\lambda^{-1}e^{-\lambda F}dv$, where F is a functional and dv is a fictitious Lebesgue measure on X and Z_λ is a normalizing constant. We consider finite dimensional case. Let X be a Riemannian manifold and F be a Morse function on X. For $\lambda > 0$, we consider a probability measure $Z_\lambda^{-1}e^{-\lambda F}dv$, where dv is the Riemannian volume. Let $|\mathrm{grad} f(x)|^2_{T_xX}$ be the square field operator defined by the gradient vector field of f on X. There have been many studies of asymptotic behavior of the spectrum of the (non-negative) generator

$-L_\lambda$ of the Dirichlet form of $\frac{1}{\lambda}\int_X |\mathrm{grad} f(x)|^2_{T_xX} e^{-\lambda F} dv$ as $\lambda \to \infty$ (Holley, Kusuoka, Stroock, Helffer, Nier, Bovier, Eckhoff, Gayrard, Klein,.....). Also, Witten gave an approach to Morse inequality by using this semiclassical analysis of the Witten Laplacian acting on differential forms.

In this talk, we consider $-L_\lambda$ with the Dirichlet boundary condition on a bounded domain of the pinned path space with the pinned Brownian motion measure ν_λ over a Lie group $SU(n)$, where the "Riemannian metric" on the space of paths is H^1-Riemannian metric. Note that ν_λ has the formal path integral representation using $F(\gamma) = \frac{1}{2}\int_0^1 |\gamma'(t)|^2 dt$ (=the energy of the path γ). The local minima of F is just the global minimum path(=minimal geodesic). We determine the limit of the spectrum of $-L_\lambda$ order $O(1)$ in terms of the eigenvalues of Hessian of F at geodesics in the domain. We also explain related previously known results (*e.g.* Eberle's result) and remaining open problems.

Curvature bound of the Dyson Brownian motion

Kohei Suzuki
(Durham University)

The Dyson Brownian motion (DBM) is an eigenvalue process of a certain Hermitian matrix valued Brownian motion introduced by Freeman Dyson in 1962, which has been one of the central subjects of random matrix theory. In this talk, we study the DBM from a geometric perspective. We show that the infinite particle Dyson Brownian motion possesses a non-negative Ricci curvature lower bound à la Bakry-Émery. As a consequence, we obtain various quantitative estimates of the transition probability of the DBM as well as the characterisation of the DBM as the gradient flow of the relative entropy in a certain Wasserstein-type space.

Liouville Quantum Gravity in Even Dimension

Lorenzo Dello Schiavo
(Institute of Science and Technology Austria)

On large classes of closed even-dimensional Riemannian manifolds M, we construct and study the Copolyharmonic Gaussian Field, i.e. a conformally invariant log-correlated Gaussian field of distributions on M. This random field is defined as the unique centered Gaussian field with covariance kernel given as the resolvent kernel of Graham–Jenne–Mason–Sparling (GJMS) operators of maximal order. The corresponding Gaussian Multiplicative Chaos is a generalization to the $2m$-dimensional case of the celebrated Liouville Quantum Gravity measure in dimension two. We study the associated random GJMS operator, the higher-dimensional analogue of the $2d$ of the random Laplacian, and show that no higher-dimensional analogue exists of the Liouville Brownian motion. Finally, we study the Polyakov–Liouville measure on the space of distributions on M induced by the copolyharmonic Gaussian field, providing explicit conditions for its finiteness and computing the conformal anomaly.

The talk is based on the joint work *Conformally invariant random fields, quantum Liouville measures, and random Paneitz operators on Riemannian manifolds of even dimension* arXiv:2105.13925, with Ronan Herry, Eva Kopfer, Karl-Theodor Sturm.

Hess-Schrader-Uhlenbrock inequality for the heat semigroup on differential forms over Dirichlet spaces tamed by distributional curvature lower bounds

Kazuhiro Kuwae
(Fukuoka University)

The notion of tamed Dirichlet space was proposed by Erbar, Rigoni, Sturm and Tamanini ('22) as a Dirichlet space having a weak form of Bakry-Émery curvature lower bounds in distribution sense. After their work, Braun ('21+) established a vector calculus for it, in particular, the space of L^2-normed L^∞-module describing vector fields, 1-forms, Hessian in L^2-sense. In this framework, we establish the Hess-Schrader-Uhlenbrock inequality for 1-forms as an element of L^2-cotangent bundles, (an L^2-normed L^∞-module), which extends the result on the Hess-Schrader-Uhlenbrock inequality under an additional condition by Braun ('21+).

Estimates for the radial part of jump-diffusion processes on manifolds and their applications

Hirotaka Kai
(Kyoto University)

Applebaum (1995) obtained a jump-diffusion process on a Riemannian manifold by the Elles-Elworthy-Malliavin construction. Since the generator of the jump-diffusion process is a kind of extension of a Lévy process on the Euclidean space, we can regard such process as a Lévy process on a Riemannian manifold. In this talk, we will show that under suitable conditions, the jump-diffusion process is transient, conservative, and has the C_0-property. In order to show these properties, I will present some estimates for the radial part of the jump-diffusion process.

Wasserstein distance on solutions to stochastic differential equations with jumps

Atsushi Takeuchi
(Tokyo Woman's Christian University)

Consider the jump processes $\{X_t\,;\,t\in[0,T]\}$ and $\{Y_t\,;\,t\in[0,T]\}$ valued in $\mathbb{R}^d$ or a Riemannian manifold M with $\dim M = d$, which are determined by stochastic differential equations with jumps. There are some approaches to construct jump processes on M, and one of them is the projection of the $O(M)$-valued process given as the solution to the Marcus-type equation with jumps. Here, $O(M)$ is the bundle of orthonormal frames on M. In this talk, let us focus on the estimates of the Wasserstein distance $d_W(X_t,\,Y_t)$. As an application, we shall give a comment on the probability laws about the subordinated Brownian motion on M the M-valued projected process.

Weak well-posedness of stochastic Volterra integral equations

Yushi Hamaguchi
(Osaka University)

In this talk, we consider the following stochastic Volterra integral equation (SVIE for short):

$$X_t = x(t) + \int_0^t K(t-s)b(X_s)\,\mathrm{d}s + \int_0^t K(t-s)\sigma(X_s)\,\mathrm{d}W_s,\ t>0,$$

where x is an $\mathbb{R}^n$-valued deterministic function, W is a d-dimensional Brownian motion, $b:\mathbb{R}^n\to\mathbb{R}^n$ is a uniformly continuous drift coefficient, $\sigma:\mathbb{R}^n\to\mathbb{R}^{n\times d}$ is a uniformly continuous diffusion coefficient satisfying the uniform ellipticity condition, and $K:(0,\infty)\to[0,\infty)$ is a completely monotone kernel which has the representation $K(t)=\int_{[0,\infty)} e^{-\theta t}\,\mu(\mathrm{d}\theta)$ for some Radon measure μ on $[0,\infty)$. Under a "balance condition" between the modulus of continuity of σ and the singularity of the kernel K, we show that the weak existence and the uniqueness in law hold for the SVIE. In order to prove this result, we introduce the following stochastic evolution equation (SEE for short) defined on a Hilbert space of functions of θ:

$$\begin{cases} \mathrm{d}Y_t(\theta) = -\theta Y_t(\theta)\,\mathrm{d}t + b(\mu[Y_t])\,\mathrm{d}t + \sigma(\mu[Y_t])\,\mathrm{d}W_t, \quad \theta\in\operatorname{supp}\mu,\ t>0,\\ Y_0(\theta) = y(\theta), \quad \theta\in\operatorname{supp}\mu, \end{cases}$$

which is a "Markovian lift" of the SVIE. By means of the generalized coupling method, we prove the weak well-posedness of the SEE, which implies the weak well-posedness of the original SVIE.

Ergodic robust maximization of asymptotic growth under stochastic factors

Josef Teichmann
(ETH Zürich)

We consider an asymptotic robust growth problem under model uncertainty and in the presence of (non-Markovian) stochastic covariance. We fix two inputs representing the instantaneous covariance for the asset process X, which depends on an additional stochastic factor process Y, as well as the invariant density of X together with Y. The stochastic factor process Y has continuous trajectories but is not even required to be a semimartingale. Our setup allows for drift uncertainty in X and model uncertainty for the local dynamics of Y. Under suitable, quite weak assumptions we are able to characterize the robust optimal trading strategy and the robust optimal growth rate. The optimal strategy is shown to be functionally generated and, remarkably, does not depend on the factor process Y. Our result provides a comprehensive answer to a question proposed by Fernholz in 2002. Mathematically, we use a combination of partial differential equation (PDE), calculus of variations and generalized Dirichlet form techniques.