Geometric Mechanics of the processes solving Schrödinger's problem

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The community of Optimal Transportation rediscovered recently "Schrödinger's (variational) problem" (1931-32) for diffusions, mostly inspired by T. Mikami's series of works initiated 20 years ago ([2]).

We shall describe the (stochastic) dynamical theory of those critical processes. They admit Lagrangian, Hamiltonian and Noetherian descriptions. As suggested originally by Schrödinger, their dynamical properties are very reminiscent of the ones expected for corresponding (but inexistent) quantum "processes". Using the terminology of modern mathematical quantum field theory, Schrödinger's inspired dynamical theory provides historically the first Euclidean approach to all aspects of elementary Q.M.

We shall emphasize particularly the Hamiltonian approach, developed recently by Qiao Huang and myself ([1]), using tools of L. Schwartz and P.A. Meyer in their "Second-order approach" to stochastic differential geometry.

[1] Q. Huang, J.-C. Z., *From second-order differential geometry to stochastic geometric mechanics,* JNLS 33 (2023), Open access

[2] T. Mikami, *Stochastic Optimal Transportation (Stochastic control with fixed marginals),* Springer 2021