Besov regularity of the density function for SDEs with super-linearly growing coefficients

Dai Taguchi
(Kansai University)

In 2010, Fournier–Printems [2] introduced a simple method for proving the existence of the density function of the time marginals of one-dimensional SDEs with linear growth (and non-smooth) coefficients. Debussche–Fournier [1] and Romito [4] extended this method to multi-dimensional case, and proved that the density function belongs to some Besov space. Their approach is based on"the one-step Euler–Maruyama scheme". On the other hand, Hutzenthalerm–Jentzen–Kloeden [3] showed that if the coefficients of SDE grow super-linearly, then the standard Euler–Maruyama scheme does not converge to a solution of the equation. In order to approximate a solution of these SDEs, several "tamed Euler–Maruyama schemes" are proposed. In particular, Sabanis [5] provided the rate of convergence for the scheme with super-linearly growing coefficients.

In this talk, inspired by these previous research, we prove the Besov regularity of the density function for a class of SDEs with super-linearly growing coefficients (for example, the stochastic Ginzburg–Landau equation in the theory of super conductivity, the Heston-3/2 volatility model in mathematical finance, and the stochastic Suart–Landau oscillator (neuronal model, hypo-elliptic case)). Our approach is based on "the one-step tamed Euler–Maruyama scheme".

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References

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