Description of fractional harmonic maps via discontinuous martingales on Riemannian submanifolds

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Harmonic maps are critical points of the energy functional defined for maps between Riemannian manifolds. It is known that these maps can be characterized through Brownian motion and martingales on manifolds. In this talk, we will consider a probabilistic characterization of harmonic maps with respect to non-local Dirichlet forms. The most typical examples of such harmonic maps are fractional harmonic maps, which are defined as harmonic maps with respect to the fractional Laplacian. To begin with, I will recall the definition of discontinuous martingales on Riemannian submanifolds. These are special cases of martingales considered in Picard (1991). Then I will elaborate on a characterization of harmonic maps for non-local Dirichlet forms via stochastic processes. In particular, from the characterization, we can obtain discontinuous martingales on Riemannian submanifolds by mapping symmetric stable processes by fractional harmonic maps. As simple applications of this approach, I will provide the Liouville-type theorem for fractional harmonic maps with values on a sphere. I will also explain the relation between the singularities of martingales on manifolds and those of harmonic maps.