Probabilistic representation for the derivative of a killed process: The multi-dimensional case

Arturo Kohatsu-Higa
(Ritsumeikan University)

In this presentation, I will explain some results (in preparation) that have been obtained with F. Antonelli (U. L'Aquila) on the representation of derivative of the killed diffusion semigroup. This representation is based on a exponential process solution of a linear stochastic equation which contains a local time term. The result can be read as an extension of the classical results of derivatives of flow diffusions. In fact, recall the classical formula for $f, b, \sigma \in C_b^1$:

$$
\begin{aligned}
X_t =& x + \int_0^t b(X_s)ds + \int_0^t \sigma^k(X_s)dW_s^k \\
\nabla_x P_T f(x) =& \nabla_x \mathbb{E}\left[f(X_T)\right] = \mathbb{E}\left[f'(X_T)\nabla_x X_T\right] \\
\nabla_x X_t =& 1 + \int_0^t \mathbf{D}b(X_s)\nabla_x X_s ds + \int_0^t \mathbf{D}\sigma^k(X_s)\nabla_x X_s dW_s^k
\end{aligned}
$$

We extend the above formula in the following sense:

$$
\nabla_x P_T f(x) = \nabla_x \mathbb{E}\left[f(X_T)1_{(\tau > T)}\right] = \mathbb{E}\left[f'(Y_T)\Psi_T\right].
$$

Here $\tau \equiv \tau^x := \inf\{s > 0; X_s \in \nabla D\}$ where D is a smooth domain and Y is the solution of an obliquely reflected stochastic differential equation with coefficients b and σ. Finally Ψ is the solution of a linear equation with diffusion coefficient $\mathbf{D}\sigma^k(Y_s)\Psi_s$, drift coefficient $\mathbf{D}b(X_s)\Psi_s$ and local time terms which have some geometrical meaning which will be discussed in the presentation. Furthermore Ψ presents jumps everytime that Y touches the boundary. These effects are "dual" to similar results about pathwise differentiation of reflected processes. These results extend the previously studied situations of the one dimensional case and the half space case with no correlation at the boundary which were obtained in joint work with D. Crisan (Imperial College).