Weak well-posedness of stochastic Volterra integral equations

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In this talk, we consider the following stochastic Volterra integral equation (SVIE for short):

$$X_t = x(t) + \int_0^t K(t-s)b(X_s)\,\mathrm{d}s + \int_0^t K(t-s)\sigma(X_s)\,\mathrm{d}W_s,\ t > 0,$$

where x is an $\mathbb{R}^n$-valued deterministic function, W is a d-dimensional Brownian motion, $b : \mathbb{R}^n \to \mathbb{R}^n$ is a uniformly continuous drift coefficient, $\sigma : \mathbb{R}^n \to \mathbb{R}^{n\times d}$ is a uniformly continuous diffusion coefficient satisfying the uniform ellipticity condition, and $K : (0,\infty) \to [0,\infty)$ is a completely monotone kernel which has the representation $K(t) = \int_{[0,\infty)} e^{-\theta t}\,\mu(\mathrm{d}\theta)$ for some Radon measure μ on $[0,\infty)$. Under a "balance condition" between the modulus of continuity of σ and the singularity of the kernel K, we show that the weak existence and the uniqueness in law hold for the SVIE. In order to prove this result, we introduce the following stochastic evolution equation (SEE for short) defined on a Hilbert space of functions of θ:

$$\begin{cases} \mathrm{d}Y_t(\theta) = -\theta Y_t(\theta)\,\mathrm{d}t + b(\mu[Y_t])\,\mathrm{d}t + \sigma(\mu[Y_t])\,\mathrm{d}W_t, & \theta \in \operatorname{supp}\mu,\ t > 0, \\ Y_0(\theta) = y(\theta), & \theta \in \operatorname{supp}\mu, \end{cases}$$

which is a "Markovian lift" of the SVIE. By means of the generalized coupling method, we prove the weak well-posedness of the SEE, which implies the weak well-posedness of the original SVIE.