Asymptotics of lowlying Dirichlet eigenvalues of Witten Laplacians on domains in pinned path groups

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In mathematical physics, there are probability measures ν_λ on infinite dimensional spaces X which are formally written as the path integral $d\nu_\lambda = Z_\lambda^{-1} e^{-\lambda F} dv$, where F is a functional and dv is a fictitious Lebesgue measure on X and Z_λ is a normalizing constant. We consider finite dimensional case. Let X be a Riemannian manifold and F be a Morse function on X. For $\lambda > 0$, we consider a probability measure $Z_\lambda^{-1} e^{-\lambda F} dv$, where dv is the Riemannian volume. Let $|\mathrm{grad} f(x)|^2_{T_x X}$ be the square field operator defined by the gradient vector field of f on X. There have been many studies of asymptotic behavior of the spectrum of the (non-negative) generator $-L_\lambda$ of the Dirichlet form of $\frac{1}{\lambda}\int_X |\mathrm{grad} f(x)|^2_{T_x X} e^{-\lambda F} dv$ as $\lambda \to \infty$ (Holley, Kusuoka, Stroock, Helffer, Nier, Bovier, Eckhoff, Gayrard, Klein,.....). Also, Witten gave an approach to Morse inequality by using this semiclassical analysis of the Witten Laplacian acting on differential forms.

In this talk, we consider $-L_\lambda$ with the Dirichlet boundary condition on a bounded domain of the pinned path space with the pinned Brownian motion measure ν_λ over a Lie group $SU(n)$, where the "Riemannian metric" on the space of paths is H^1-Riemannian metric. Note that ν_λ has the formal path integral representation using $F(\gamma) = \frac{1}{2}\int_0^1 |\gamma'(t)|^2 dt$ (=the energy of the path γ). The local minima of F is just the global minimum path(=minimal geodesic). We determine the limit of the spectrum of $-L_\lambda$ order $O(1)$ in terms of the eigenvalues of Hessian of F at geodesics in the domain. We also explain related previously known results (*e.g.* Eberle's result) and remaining open problems.